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NEW FORMULA FOR THE EVALUATION OF ACCURACY OF ANGULAR MEASURMENTS AT THE POINTS OF TRIANGULATION

معادلة جديدة لحساب معيار الدقة للقياس الزاوي لنقط شبكات المثلثات

BY

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الملخص العربي

إن الاستخدام الشائع للطرق المستخدمة في قياس الزوايا الأفقية بكل تركيباتها في شبكات المثلثات في منتهى الأهمية في أعمال الرفع المساحي . ونظراً للاهتمام الشديد في الآونة الأخيرة بالدقة ومعاييرها تم في هذا البحث استنتاج معادلة رياضية تسهل الحصول على الدقة المطلوبة , حيث أن الطرق المستخدمة في القياس الزاوي للاتجاهات حول نقطة من نقط شبكات المثلثات وتقنياتها لا تسهل الحصول علي هذه الدقة. إن المعادلة التي تم استنتاجها في هذا البحث معادلة بسيطة وسهلة الاستخدام وتتطلب حسابات اقل خصوصاً عندما يكون عدد الاتجاهات حول نقطة المثلثات كبير(من ستة إلى تسعة اتجاهات), كما أن هذه المعادلة تعطي نتائج ممتازة بالمقارنة بالمعادلات الأخرى .علاوة على ذلك فان المعادلة المستنتجة في هذا البحث ملائمة جداً ومن المفضل استخدامها في التطبيقات العملية.

Abstract

For the widely used method of measuring angles in all combinations in triangulation nets, as well as for other methods of angular measurements there is no techniques enabling to find the value of mean square errors for each sight taken separately. For the angular measurements carried out by the method of rounds there was a formulae deduced for the first time , but it revealed to be not too easy to use.

In this paper, there is a new formula deduced herein, which is much more simple. Moreover, it requires less computations, specially when the number of directions observed is great($n \geq 6—9$). For the same directions this formula give very good results comparing with other formulae. Since this formula deduced in this paper is more convenient and simple, it should be preferred in practice.

Key words: Accuracy, triangulation, angular measurements.

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I. Introduction

As it is known, the triangulation requires that the results of angular measurements at a station (point) and their adjustment be represented as a series of equal accuracy sights having the same weight P_j = constant, j = 1,2,3,........,n.

Traditionally the weight of adjustment sights measured from a station is assumed being equal to the number of the repetition or proportional to this number. For instance, when the directions are measured in all combinations, it is taken as:

$$P_j = mn. \qquad (1)$$

And for the observations performed by rounds,

$$P_j = m. \qquad (2)$$

Where:

m is the number of rounds, n is the number of points sighted.

Formulae (1) and (2) do not reflect real conditions under which the observations were carried out.

Many factors are not taken into account in these assumptions, e.g. different visibility along different sights, different amplitudes of oscillation of sighting targets observed in the field of view, different influence of local fields of lateral refraction along different sights and so on.

That is why the assumptions mentioned above are not able to give a correct idea on real weight of each sight taken separately.

More reliable data on the weights of adjusted sights could be obtained if it is computed with the use of results of direct measurements, i.e. by means of:

$$P_j = 1/M_j^2 \qquad (3)$$

Where:

M_j is the mean square error of j-th adjusted sights, j = 1,2,3,..........,n.

For the widely used method of measuring angles in all combinations, as well as for other methods of angular measurements there are no techniques enabling [1,2] to find the value of M_j for each sight taken separately.

For the angular measurements carried out by the method of rounds there is a formulae deduced for the first time, but it revealed to be not too easy to use.

In this paper a new formula demanding less computations, is deduced. Also an example of accuracy evaluation using this formulae for the case of 2nd order triangulation is presented.

II. Theoretical Grounds

When measuring angles by the method of rounds, the direction are observed at different settings of the theodolite circle. Between the rounds the circle is off-set by an angle.

$$\beta = 180^\circ / m + i, \qquad (4)$$

where:

m is the number of rounds,

i being the scale division of the circle.

During the treatment of observation results the misclosure of rounds are distributed evenly between all the directions. Then all the sights are reduced to the initial (first) one which is taken as zero.

Let N_1'', N_2'', N_3'', ..., N_n'' be the directions observed in a round with certain setting of the circle.

After reducing them to the initial sight we have:

$$N_1' = N_1'' - N_1'' = 0;$$

$$N_2' = N_2'' - N_1'' = 1.2;$$

$$N_3' = N_3'' - N_1'' = 1.3; \qquad (5)$$

....

$$N_n' = N_n'' - N_1'' = 1.n;$$

The sights adjusted at station N_j (j = 1,2, ...,n) and reduced to the initial one could be computed as the mean value for m rounds:

$$N_1 = 0^\circ 00' 00,00'';$$

$$N_2 = [1.2] = \frac{1}{m}\sum 1.2;$$

$$N_3 = [1.3] = \frac{1}{m}\sum 1.3; \qquad (6)$$

....

$$N_n = [1.n] = \frac{1}{m}\sum 1.n.$$

Adjusted values N_j could be found in another way if the correction δ_j for all the directions observed in each round are found .

Instead of equation (6) we could write:

$$N_1 = N_1' + \delta_1;$$

$$N_2 = N_2' + \delta_2;$$

$$N_3 = N_3' + \delta_3; \qquad (7)$$

....

$$N_n = N_n' + \delta_n.$$

Where:

N_j' are the directions observed in a round.

The formulae enabling to compute δ_j could be obtained by rewriting (6) allowing for (7):

$N_1 = 0°00'00,00''$;

$N_2 = [1.2] = (N_2' + \delta_2) - (N_1' + \delta_1)$;

$N_3 = [1.3] = (N_3' + \delta_3) - (N_1' + \delta_1)$; (8)

....

$N_n = [1.n] = (N_n' + \delta_n) - (N_1' + \delta_1)$.

Differences $\delta_j - \delta_1$ will be hereinafter denoted as $\upsilon_{i,j}$.

$\delta_2 - \delta_1 = \upsilon_{1.2}$;

$\delta_3 - \delta_1 = \upsilon_{1.3}$;

$\delta_4 - \delta_1 = \upsilon_{1.4}$; (9)

....

$\delta_n - \delta_1 = \upsilon_{1.n}$.

These values $\upsilon_{1.2}, \upsilon_{1.3}, \upsilon_{1.4}, \ldots, \upsilon_{1.n}$

in a round could also be expressed as differences between adjusted sights (mean values computed for m rounds) and directions observed in this round:

$\upsilon_{1.2} = N_2 - N_2' = [1.2] - 1.2$;

$\upsilon_{1.3} = N_3 - N_3' = [1.3] - 1.3$;

$\upsilon_{1.4} = N4 - N4' = [1.4] - 1.4$; (10)

....

$\upsilon_{1.n} = Nn - Nn' = [1.n] - 1.n$;

After summing up equations (9) and after adding $n\delta_1$ to the sums of right and left parts, we obtain:

$$\delta_1 + \delta_2 + \delta_3 + \ldots + \delta_n = n\delta_1 + \upsilon_{1.2} + \upsilon_{1.3} + \ldots + \upsilon_{1.n} \quad (11)$$

where:

n is the number of directions observed and δ_1 is the correction to the initial sight.

Using formulae (11) and taking into account that, in each single round the sum of corrections δj is equal to zero:

$$\sum \delta_j = 0. \quad (12)$$

The formula enabling to compute the correction δ_1 to the initial direction observed in each round can now be attained.

$$\delta_1 = -\frac{1}{n}\sum_{j=2}^{n}\upsilon_{1.j}. \quad (13)$$

After computing corrections $\upsilon_{1.2}$, $\upsilon_{1.3}$,..........., $\upsilon_{1.n}$ by formulae (10) and the correction δ_1 to the initial direction by formula (13) , corrections δj to all the observed directions in a given round could then be found with the use of formulae (9):

$$\delta_1 = -\frac{1}{n}\sum_{j=2}^{n}\upsilon_{1,j} \quad ;$$

$$\delta_2 - \delta_1 + \upsilon_{1,2} ;$$

$$\delta_3 - \delta_1 + \upsilon_{1,3} ; \qquad (14)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\delta_n - \delta_1 + \upsilon_{1,n} .$$

Corrections δj are to be computed for all the directions observed in each round.

It becomes possible to find the mean square error averaged from all n directions of a measurement having unit weight:

$$\mu = \sqrt{\frac{\sum\left[\delta_j^2\right]}{(m-1)(n-1)}} \qquad (15)$$

where:

$\sum\left[\delta_j^2\right]$ is the sum of squares of δj computed with the use of all n directions observed in all m rounds, the number of redundant measurements in this case is equal to mn – (n-1+ m).

for the evaluation of accuracy of a direction, the number of redundant measurements is to be taken in n times less than for all n directions taken together. This means that in case of a direction taken separately, the formula enabling to compute the mean square error of a measurement having unit weight, will look like follows:

$$\mu_j = \sqrt{\frac{n\sum\delta_j^2}{(m-1)(n-1)}} \qquad (16)$$

Mean square error of each adjusted direction (averaged from m rounds) could be found from:

$$M_j = \frac{\mu}{\sqrt{m}} = \sqrt{\frac{n\sum\delta_j^2}{m(m-1)(n-1)}} \qquad (17) .$$

In Table 1 is given an example of computation of mean square errors of all observed directions with the use of formula (17).

In our previous publication [3] another formula enabling to calculate mean square errors M_j, was deduced :

$$M_j = \sqrt{\frac{(n-2)\left[\upsilon_{j1}^2\right]-\left[\upsilon_{1k}^2\right]}{m(m-1)(n-1)(n-2)}} \qquad (18)$$

The results of computations with the use of formulae (17) and (18) for the example mentioned above, are shown in Table 2.

III. Conclusions

A new formula (17) deduced herein, is much more simple than formula (18) proposed earlier.

Moreover, it requires less computations, specially when the number of directions observed is great ($n \geq 6—9$). For the same directions both formulae give the same results, which means that they are correct. Since formula (17) is more convenient and simple, it should be preferred in practice.

IV. References

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Table 1 Adjustment and accuracy evaluation of directions observed by the method of rounds at the 2nd order triangulation point (**Initial direction, $N_1 = 0^0\,00'\,00.00''$; n = 4 , m = 12)**

Round i	N2=1.2			N3=1.3			N_4=1.4			$\sum \upsilon_{1j}$	$\delta_1 = -\sum \frac{\upsilon_{1j}}{n}$	Control
No.	$63^0\,15'$	$\upsilon_{1.2}$	δ_2	$109^0\,47'$	$\upsilon_{1.3}$	δ_3	$186^0\,34'$	$\upsilon_{1.4}$	δ_4			$\sum \delta_j = 0$
1	44.0"	-1.32"	-0.50"	24.1"	+0.12"	-0.70"	47.2	-1.86"	+1.04"	+3.30"	-0.82"	+0.02"
2	45.8	+0.48	+0.87	26.7	-2.48	-1.13	51.5	-2.44	-1.09	-5.40	+1.35	0.00
3	44.3	-1.02	+0.57	22.4	+1.82	+1.37	50.1	-1.04	-1.49	+1.80	-0.45	0.00
4	43.0	-2.32	+1.10	24.8	-0.58	-1.80	45.9	+3.16	-1.94	+4.90	-1.22	+0.02
5	46.7	+1.38	+0.17	27.3	-3.08	-1.53	50.8	-1.74	-0.19	-6.20	+1.55	0.00
6	44.0	-1.32	+0.34	22.5	+1.72	+0.74	48.2	+0.86	-0.12	+3.90	-0.98	-0.02
7	44.6	-0.72	+0.54	23.9	+0.32	+0.14	49.4	-0.34	-0.52	+0.70	-0.18	-0.02
8	46.0	+0.68	-1.03	23.0	+1.22	+0.87	48.2	+0.86	-0.51	+1.40	-0.35	0.00
9	46.4	+1.08	-0.88	23.3	+0.92	+1.12	49.7	-0.64	-0.44	-0.80	+0.20	0.00
10	47.8	+2.48	-1.36	25.4	-1.18	-0.06	49.9	-0.84	+0.28	-4.50	+1.12	-0.02
11	44.9	-0.42	+0.54	24.0	+0.22	+0.34	50.2	-1.14	-1.02	-0.50	+0.12	-0.02
12	46.4	+1.08	-1.43	23.2	+1.02	+0.67	47.6	+1.46	-1.11	-1.40	-0.35	0.00
$[1.j]$	45.32			24.22			49.06					
$\sum 1.j$		+0.06	-0.07		+0.04	+0.03		+0.02	0.01	0.00	-0.01	-0.04
$\sum \delta_j^2$			8.9993			12.3713			11.3789		9.1349	
M_j			0.30			0.35			+0.34		0.30	

Table 2

Directions	**1**	**2**	**3**	**4**
M_j by (17)	0.30″	0.30″	0.35″	0.34″
M_j by (18)	0.29″	0.29″	0.37″	0.34″