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DAMPING MATERIAL FOR STABILIZATION OF PANEL FLUTTER AT HIGH SUPERSONIC SPEED

إستخدام المواد الكابحة للرفرفة للوصول إلى حالة الإتزان للألواح الواقعة فى حالة رفرفة على السرعات الأعلى من سرعة الصوت.

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الخلاصة

تتعرض الألواح الخارجية المكونة للكساء الخارجى للمركبات التى تطير بسرعات أعلى من سرعة الصوت فى الغلاف الخارجى لما يسمى بالإهتزازات المثارة ذاتيا من النوع ذو الترددات المحدودة. إن تطور الطرق المستخدمة فى التحكم فى هذه الظاهرة مطلوب وهو الهدف من هذه الدراسة.

وفى هذه الدراسة تم إستخدام المعادلات اللاخطية للألواح بموائمة كل من نظرية الإنحناءات الكبيرة فى الألواح لفان كارمن والنظرية الإيروديناميكية الخطية لسيال الهواء المنتظم على سطح الألواح لتناسب الحالة تحت الدراسة وهى عبارة عن لوح من الديور الومنيوم مثبت عليه طبقة من مادة ماصة للطاقة المسببة للإهتزازات (Damping Material) . وفى الحالة المتقدمة يتم تغطية المادة الماصة للطاقة المسببة للإهتزازات بلوح أقل سمكا من اللوح نفسه وذلك لتقييد حركة المادة الملصقة على اللوح مما ينتج عنه ما يسمى بالـ (Shear Damping) وفى جميع الأحوال يتم التأكد من جودة إلتصاق المادة بسطح اللوح واللوح المقيد.

وفى هذه الدراسة تم إيضاح تأثير وقدرة المواد الماصة للطاقة المسببة لإهتزازات الألواح على إيقاف رفرفة الألواح عند ظروف عملها العادية وذلك من خلال مقارنة النتائج الخاصة بظاهرة الرفرفة للألواح بعد المعالجة وقبلها. وقد أثبتت النتائج أيضا فى هذه الدراسة أن قدرة النموذج الثانى بإستخدام لوح مقيد لعمل المادة الملصقة بالتحكم فى إيقاف ظاهرة الرفرفة للألواح فى ظروف عملها العادية أعلى من قدرة النموذج الأول.

Abstract

Exterior panels forming the exterior skin of flight vehicles traveling through the atmosphere at high supersonic speed are often susceptible to the occurrence of limit-cycle type self- excited vibrations called flutter. The development of methods for the prevention and control of flutter, without adversely affecting the weight and cost, is desirable and it is the objective of this work by using damping materials.

The nonlinear equations of motion for panel flutter using Von-Karman's large deflection plate theory are modified to include damping material layers. At first, a viscoelastic damping layer is considered to be perfectly bonded to the surface of the plate to investigate the extension damping treatment. Secondly, the viscoelastic damping layer over the plate is constrained by a third layer called constrained layer to investigate the shear damping treatment.

The two types of damping treatment proved to be able to prevent panel flutter successfully at prescribed operating condition. But for the same panel weight, the constrained damping layer technique is more efficient than the free damping layer technique.

KEY WORDS

Panel Flutter, Aerodynamic Loading, Aerodynamic Damping, Structural Damping, Cavity Effect, Thermal Effect, Damping Material, Extension Damping Treatment, and Shear Damping Treatment.

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I. Introduction:

Panel flutter results from the interaction between the panel and the flow pressure forces brought about by the panel motion. This causes a loss of the stability of the panel in its un-deformed shape, so that any disturbance applied to it leads to oscillations of growing amplitude. Thus, it is a self excited oscillation resulting from the dynamic instability of the aerodynamic, inertia, and elastic forces of the system. This growth is limited, however, by the membrane tension stresses induced in the panel by the flutter motion itself. The result of this self-limiting action is a sustained oscillation of constant amplitude, called limit-cycle motion.

Due to the complexity of panel flutter, most theoretical studies make use of simplified assumptions, see Ref. [10] and [15]. However these assumptions are usually so restrictive that the theoretical model does not accurately represent realistic conditions. In fact it is found that the application of the exact aerodynamic theory does not remove the discrepancies that presently exist between theory and experiment for flutter of stressed panels. The inclusion of structural damping is found to have a large effect in some instances and can tend to eliminate some of the differences.

There are certain unifying features common to all aeroelastic problems which provide a convenient framework for introducing and classifying the entire subject. These features include the casting of the aeroelastic equations in an operator form and the generalized solution of such operator equations, see Ref. [1]. Investigation of the theoretical foundations, methods of analysis for treating linear aeroelastic models, their nonlinear counterparts, and the requisite aerodynamic theory were discussed, with the three levels of approximation to the motion dependent aerodynamic pressures on an oscillating panel, see Ref. [2]. The simplest of these and consequently the most widely used is the so called " piston theory, see Ref. [3].

Once the mathematical model has been established as in Ref. [10] and [15], the methods of solution are required to investigate the parameters variations. Fortunately, as far as the treatment of the panel flutter of a finite plate is concerned, the Galrkin's method gives qualitatively " correct results. Von Karman's large deflection plate theory and quasi-steady aerodynamic theory have been employed. Galerkin's technique has been used in the space variables and the ordinary differential system obtained is solved by an asymptotic expansion using method of multiple time scales. The results obtained show that, as a first approximation, the amplitude of the limit cycle depends only upon the fundamental parameter (non-dimensional aerodynamic loading), the aspect ratio, and the damping parameter (including structural damping effect). The pre-flutter panel motion and the motion during flutter were studied in detail, as shown in Ref. [4].

Flutter oscillations rarely cause immediate failure of the panel, but they may produce fatigue failure after a sufficient period of time. The need to prevent this occurrence, either by suppressing flutter entirely or by limiting the severity of the panel motion, often becomes the critical design criterion that determines the required thickness (or more generally the stiffness) of the panel. Many parameters govern the resonance fatigue behavior including the detail design, the skin thickness and materials, the stiffener configurations and the damping of the structure. Damping can be introduced when needed into a structure without creating adverse effects in design, weight, or cost.

II. Problem Formulation

The plate under consideration with cavity is shown in Fig. 1. The dimensions and the properties of the plate are given in Table2. The axes are taken to be such that the x-, y- axes are in the plane of the plate passing through its reference plane (z=0), while the z-axis is positive upwards. Under the assumptions that plate thickness is small in comparison with smallest lateral dimension, which is the case in most practical applications, the Kirchhoff's hypothesis may be assumed to be valid. With this assumption the in-plane displacements "u", "v" and the transverse deflection "w" at an arbitrary point of the plate in the x, y, and z directions will be as given in Ref. [10] and [15]: The final aero-elastic equations of the plate in a non-dimensional form is found to be:

The U-equation is given as:

$$\bar{u}_{,\xi\xi}+d_1f^2\bar{u}_{,\eta\eta}+d_2f\bar{v}_{,\xi\eta}+\frac{h}{a}\{\bar{w}_{,\xi}\bar{w}_{,\xi\xi}+d_1f^2\bar{w}_{,\xi}\bar{w}_{,\eta\eta}+d_2f^2\bar{w}_{,\eta}\bar{w}_{,\xi\eta}\}=0 \quad (1\text{-a})$$

The V-equation is given as:

$$d_1\bar{v}_{,\xi\xi}+f^2\bar{v}_{,\eta\eta}+d_2f\bar{u}_{,\xi\eta}+\frac{h}{b}\{f^2\bar{w}_{,\eta}\bar{w}_{,\eta\eta}+d_1\bar{w}_{,\eta}\bar{w}_{,\xi\xi}+d_2\bar{w}_{,\xi}\bar{w}_{,\xi\eta}\}=0 \quad (1\text{-b})$$

The W-equation is given as:

$$\frac{1}{\pi^4}\{\bar{w}_{,\xi\xi\xi\xi}+2f^2\bar{w}_{,\xi\xi\eta\eta}+f^4\bar{w}_{,\eta\eta\eta\eta}\}\pm\{\bar{n}_\eta^{(AL)}+\bar{n}_\eta^{(AT)}\}\pm\{\bar{n}_\xi^{(AL)}+\bar{n}_\xi^{(AT)}\}+\lambda_a\bar{w}_{,\xi}+\lambda_d\bar{w}_{,\tau}+\lambda_c\int_0^1\int_0^1\bar{w}\,d\eta\, d\xi+\bar{w}_{,\tau\tau}-\frac{1}{\pi^4}\{12\frac{a}{h}[\bar{u}_{,\xi}\bar{w}_{,\xi\xi}+\nu f^2\bar{u}_{,\xi}\bar{w}_{,\eta\eta}+(1-\nu)f^2\bar{u}_{,\eta}\bar{w}_{,\xi\eta}+f^3\bar{v}_{,\eta}\bar{w}_{,\eta\eta}+\nu f\bar{v}_{,\eta}\bar{w}_{,\xi\xi}+(1-\nu)f\bar{v}_{,\xi}\bar{w}_{,\xi\eta}]+6[\bar{w}^2_{,\xi}\bar{w}_{,\xi\xi}+f^4\bar{w}^2_{,\eta}\bar{w}_{,\eta\eta}+f^2\bar{w}^2_{,\eta}\bar{w}_{,\xi\xi}+f^2\bar{w}^2_{,\xi}\bar{w}_{,\eta\eta}+f^2\bar{w}_{,\xi}\bar{w}_{,\eta}\bar{w}_{,\xi\eta}]\}=0 \quad (1\text{-c})$$

Where, $d_1=(1-\nu)/2$ and $d_2=(1+\nu)/2$

The form of these equations is general see Ref [1] and Ref. [9]. Galerkin's method is then used to reduce the obtained equations of motion to a set of nonlinear ordinary differential equations having the non-dimensional time variable as an independent variable. The displacement u, v, and the deflection w are expanded in the form of a generalized double series of modes. These modes satisfy the appropriate geometric boundary conditions of the plate, thus:

$$\bar{u}^o(\xi,\eta,\tau)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\bar{U}_{mn}(\tau)X^{(u)}_m(\xi)Y^{(u)}_n(\eta) \quad (2\text{-a})$$

$$\bar{v}^o(\xi,\eta,\tau)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\bar{V}_{mn}(\tau)X^{(v)}_m(\xi)Y^{(v)}_n(\eta) \quad (2\text{-b})$$

$$\bar{w}^o(\xi,\eta,\tau)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\bar{W}_{mn}(\tau)X^{(w)}_m(\xi)Y^{(w)}_n(\eta) \quad (2\text{-c})$$

where $X(\xi)$ and $Y(\eta)$ are the modal functions that satisfy the boundary conditions imposed on u, v, or w in the ξ and η direction respectively.

$$A1^{mn}_{ij}\bar{U}_{mn}+A2^{mn}_{ij}\bar{V}_{mn}+A3^{mnrs}_{ij}\bar{W}_{mn}\bar{W}_{rs}=0 \quad (3\text{-a})$$

$$B1^{mn}_{ij}\bar{V}_{mn}+B2^{mn}_{ij}\bar{U}_{mn}+B3^{mnrs}_{ij}\bar{W}_{mn}\bar{W}_{rs}=0 \quad (3\text{-b})$$

$$C1^{mn}_{ij}\bar{W}_{mn,\tau\tau}+\lambda_d C1^{mn}_{ij}\bar{W}_{mn,\tau}+\lambda_a C2^{mn}_{ij}\bar{W}_{mn}+\lambda_c C3^{mn}_{ij}\bar{W}_{mn}+C4^{mn}_{ij}\bar{W}_{mn}\pm\{n_\xi^{(AL)}+n_\xi^{(AT)}\}C5^{mn}_{ij}\bar{W}_{mn}\pm\{n_\eta^{(AL)}+n_\eta^{(AT)}\}C6^{mn}_{ij}\bar{W}_{mn}+C7^{mnlp}_{ij}\bar{U}_{mn}\bar{W}_{lp}+C8^{mnlp}_{ij}\bar{V}_{mn}\bar{W}_{lp}+C9^{mnlprs}_{ij}\bar{W}_{mn}\bar{W}_{lp}\bar{W}_{rs}=0 \quad (3\text{-c})$$

The vectors $U_{mn}(\tau)$ and $\bar{V}_{mn}(\tau)$ are computed from equations (3-a) and (3-b) algebraically and then substituted pinto equation (3-c) to yield the Duffing-type equation in the following form:

$$C1^{mn}_{ij}\bar{W}_{mn,\tau\tau}+\lambda_d C2^{mn}_{ij}\bar{W}_{mn,\tau}+C3^{mn}_{ij}\bar{W}_{mn}\pm\{n_\xi^{(AL)}+n_\xi^{(AT)}\}C4^{mn}_{ij}\bar{W}_{mn}\pm\{n_\eta^{(AL)}+n_\eta^{(AT)}\}C5^{mn}_{ij}\bar{W}_{mn}+\lambda_a C6^{mn}_{ij}\bar{W}_{mn}+\lambda_c C7^{mn}_{ij}\bar{W}_{mn}-T^{mnlprs}_{ij}\bar{W}_{mn}\bar{W}_{lp}\bar{W}_{rs}=0 \quad (4)$$

This tensorial nonlinear ordinary differential equation represents a set of (ixj) nonlinear ordinary differential equation which can not be solved exactly.

To complete the solution, the boundary conditions for the plate must be specified. For the purpose of comparison with the result obtained by the experimental report by Ref. [6], the boundary conditions are specified to be clamped all around with no in-plane movements, so that the out of plane boundary condition is fully-clamped and the in-plane boundary condition is edge-fixed, and these conditions are written as:

$w=w_{,\xi}=u^o=v^o=0$ at both x=0 & x=1 (5-a)

$w=w_{,\eta}=u^o=v^o=0$ at both y=0 & y=1 (5-b)

And the modal functions are then given by:

$X^{(u)}{}_m(\xi)=X^{(v)}{}_m(\xi)=\sin(m\pi\xi)$ (6-a)

And $Y^{(u)}{}_n(\eta)=Y^{(v)}{}_n(\eta)=\sin(n\pi\eta)$ (6-b)

While: $X^{(w)}{}_m(\xi)=\cosh\alpha_m\xi-\cos\alpha_m\xi-\gamma_m(\sinh\alpha_m\xi-\sin\alpha_m\xi)$ (6-c)

and: $Y^{(w)}{}_n(\eta)=\cosh\alpha_n\eta-\cos\alpha_n\eta-\gamma_n(\sinh\alpha_n\eta-\sin\alpha_n\eta)$ (6-d)

where α_m and γ_m are the coefficients for the m^{th} flexural mode and can be calculated from the following equation:

$\cosh\alpha_m \cos\alpha_m=1$ (7-a)

$\cosh\alpha_m-\cos\alpha_m\xi-\gamma_m(\sinh\alpha_m-\sin\alpha_m)=0$ (7-b)

The values of these coefficients are given in Table 1, see also Ref. [13]. Once the displacement modal functions are specified for the prescribed boundary conditions, the coefficients of integration represented by the matrices A1 to A3, B1 to B3, C1 to C9, and T can be calculated numerically, see Ref [13]. The number of the terms in the assumed displacement series solution given by (2-a), (2-b), and (2-c) are arbitrary.

Since an exact solution of equation (4) is not available, an approximate solution will be obtained by the method of harmonic balance. This method is used to seek a periodic solution. In the frequency domain the differential operator $d/d\tau$ is replaced by $j\omega$ and, consequently, the system of nonlinear differential equations is converted to a set of nonlinear algebraic equations. When this set has a solution with real positive values of frequency and amplitude and real values of the phase angles, it indicates the occurrence of limit-cycle oscillations of the specified form. Thus one seeks a solution of the following form:

$\overline{W}_{mn}(\tau)=A_{mn}\sin(\omega\tau+\Phi_{mn})$ (8-a)

$\overline{W}_{mn}=A_{mn}[\cos\Phi_{mn}+j\sin\Phi_{mn}]\sin\omega_\tau$ (8-b)

$\overline{W}_{mn,\tau}=\omega A_{mn}[j\cos\Phi_{mn}-\sin\Phi_{mn}]\sin\omega_\tau$ (8-c)

$\overline{W}_{mn,\tau\tau}=-\omega^2A_{mn}[\cos\Phi_{mn}+j\sin\Phi_{mn}]\sin\omega_\tau$ (8-d)

$\overline{W}_{mn}\overline{W}_{rs}\overline{W}_{pq}=A_{mn}A_{rs}A_{pq}\{[\cos\Phi_{mn}\cos\Phi_{rs}\cos\Phi_{pq}-\sin\Phi_{mn}\sin\Phi_{rs}\cos\Phi_{pq}-\cos\Phi_{mn}\sin\Phi_{rs}\sin\Phi_{pq}-\sin\Phi_{mn}\cos\Phi_{rs}\sin\Phi_{pq}]-j[\sin\Phi_{mn}\sin\Phi_{rs}\sin\Phi_{pq}-\cos\Phi_{mn}\cos\Phi_{rs}\sin\Phi_{pq}-\sin\Phi_{mn}\cos\Phi_{rs}\cos\Phi_{pq}-\cos\Phi_{mn}\sin\Phi_{rs}\cos\Phi_{pq}]\}\sin^3\omega_\tau$ (8-e)

Where,

$\sin^3\omega_\tau=\frac{1}{4}(3\sin\omega_\tau+\frac{3}{4}\sin 3\omega_\tau)$ (8-f)

The second term which is of higher harmonic will be dropped out and:

$\sin^3\omega_\tau=\frac{3}{4}\sin\omega_\tau$ (8-g)

Substituting $\overline{W}_{mn}$ from equations (8) into the Duffing type equation (4) and equating separately the real and imaginary parts of the equations, the set of 2(ixj) nonlinear algebraic equations in the unknown variables given by (ixj) amplitudes, ((ixj)-1) phase angles, and the flutter frequency (ω_f) is obtained. These equations are solved numerically by the modified Newton-Raphson algorithm.

III. Flutter Control by Damping Treatment

In Ref. [10] and [15], it is found that flutter occurs at a certain dynamical pressure. This critical dynamical pressure is function of the speed and altitude of flight (pressure, temperature, and density), the Mach number, and the rigidity of the plate. If the aircraft has to fly around this flight condition, it is necessary to find out some way to suppress the flutter or to prevent its occurrence. One obvious way to prevent flutter at this flight condition is to increase the rigidity of the plate by increasing its thickness or by choosing another material with higher modulus of elasticity. In fact this technique will lead to excessive weight and cost. Another technique, which is used in this chapter, to prevent flutter is the application of a damping layer to the surface of the plate. Ideally, the material used for dampers should eliminate vibrations of all possible frequencies occurring in the plate at all temperatures of operation. In addition, the material should be strain insensitive and have a low density. However, the successful application of such technique depends a great deal on three important factors. These are the knowledge of how the properties of the damping materials vary with the environment to which they are subjected, a good understanding of the dynamics of the structure to which the damping material is to be applied, and finally, how to design a material in damping treatment configuration for the desired performance. In fact, there are mainly two types of the surface damping treatments called extensional damping and shear damping, which will be illustrated separately. The damping material in the case of extensional damping is usually stiff, but not stiffer than the base plate, while in case of shear damping it is soft, such as rubber like materials. The rubber like materials, often have a far greater damping capability and are more linear with respect to strain amplitude. On the other hand, they are usually temperature sensitive.

III.1 Damping Material Data

The viscoelastic material used is the 3M ISD # 113 viscoelastic polymer. This material is manufactured into two types of tapes named SJ 2040 X and SJ 2056 X. Both of them consist of two layers of 3M ISD # 113 viscoelastic polymer material, with polyester interleaving to afford dimensional stability during application. SJ 2040 X is available in rolls, while SJ 2056 X is sheet version of the same material with double liner. These two forms of high energy dissipative polymer, which when properly incorporated into a constrained layer damping configuration, can afford excellent control of the resonance induced vibration problems. The dynamic performance, however, of the ISD # 113 is well characterized and typical data for this viscoelastic polymer is shown in Fig. 2. To find out the loss factor and the shear modulus first, the frequency range must be known. From, the frequency value (horizontal lines) and the operating temperature (isothermal lines), the point of intersection can be located on the curve. From this point going vertically up and/or down until crossing the modulus and loss factor curves, their values are given on the left vertical scale. Physical data such as thicknesses, densities, and availability of these types are given in Table2.

III.2 Extension Damping Treatment

In this technique, a viscoelastic damping layer is bonded perfectly to the surface of the plate see Fig. 3. As the plate vibrates in bending, the damping layer will deform principally in extension and compression in planes parallel to the plate surface. Actually this type of treatment is clear and simple and can be applied directly to the plate in service where these layers are available with its bonding material in the market. The viscoelastic damping layer material must be chosen carefully to perform efficiently in the range of the operating temperature of the plate. The feature of this type of damping is that the system loss factor increases with the thickness, elasticity modulus and with the loss factor of the viscoelastic layer. The damping effect is obtained due to internal losses in the viscoelastic layer resulting from the alternating extensional and compressional strains. As might be expected, the performance of the free layer damping treatment depends strongly on the method of attachment to the plate. This adhesive layer must be as thin and stiff as possible. The plate with the damping layer is shown in Fig. 15. The neutral plane of the plate will be shifted a distance, m, from its original position as a result of the bonded layer. This shift can be determined by recognizing that the net thrust in the x- and y-directions in the absence of the in-plane forces must be zero, i.e.

$$\sum_k \int \sigma_x^{(k)} dz = 0 \text{ and } \sum_k \int \sigma_y^{(k)} dz = 0 \qquad (9)$$

where under pure bending,

$$\sigma_x^{(k)} = -G_{(k)} z w_{,xx} \text{ and } \sigma_y^{(k)} = -G_{(k)} z w_{,yy} \qquad (10)$$

where: $G_{(k)} = \frac{E_{(k)}}{2(1-\nu_{(k)})}$ is the shear modulus of elasticity of the k^{th} layer.

After performing the integration in (9) and simplifying, the following expression for the neutral plane shift is given as:

$$M = 1 - \left\{\frac{E_2 h_2^2 (1+\nu_2)}{E_1 h_1^2 (1+\nu_1)}\right\} / \left\{1 + \frac{E_2 h_2 (1+\nu_2)}{E_1 h_1 (1+\nu_1)}\right\} \qquad (11)$$

Once the neutral plane shift, m, is calculated then from the strain-displacement relations, the stress resultants N_x, N_y, N_{xy} and the resultant couples M_x, M_y, M_{xy} for the composite plate can be determined by the same procedures as in Ref [10] and [15]. It is found in this case that:

$$N_x = Q_1 \frac{E_1 h_1}{1-\nu_1^2} [u^o_{,x} + (1/2) w^2_{,x}] + Q_2 \frac{\nu_1 E_1 h_1}{1-\nu_1^2} [v^o_{,y} + (1/2) w^2_{,y}] - Q_3 \frac{E_1 h_1^2}{1-\nu_1^2} [w_{,xx}] - Q_4 \frac{\nu_1 E_1 h_1}{1-\nu_1^2} [w_{,yy}] \qquad (12\text{-a})$$

$$N_y = Q_1 \frac{E_1 h_1}{1-\nu_1^2} [v^o_{,x} + (1/2) w^2_{,y}] + Q_2 \frac{\nu_1 E_1 h_1}{1-\nu_1^2} [u^o_{,y} + (1/2) w^2_{,x}] - Q_3 \frac{E_1 h_1^2}{1-\nu_1^2} [w_{,yy}] - Q_4 \frac{\nu_1 E_1 h_1^2}{1-\nu_1^2} [w_{,xx}] \qquad (12\text{-b})$$

$$N_{xy} = Q_5 G_1 h_1 [u^o_{,y} + v^o_{,x} + w_{,x} w_{,y}] - 2 Q_6 G_1 h_1^2 [w_{,xy}] \qquad (12\text{-c})$$

$$M_x = Q_3 \frac{E_1 h_1^2}{(1-\nu_1^2)} [u^o_{,x} + (1/2) w^2_{,x}] + Q_4 \frac{\nu_1 E_1 h_1^2}{(1-\nu_1^2)} [v^o_{,y} + (1/2) w^2_{,y}] - Q_7 \frac{E_1 h_1^3}{12(1-\nu_1^2)} [w_{,xx}] + Q_8 \frac{\nu_1 E_1 h_1^3}{12(1-\nu_1^2)} [w_{,yy}] \qquad (12\text{-d})$$

$$M_y = Q_3 \frac{E_1 h_1^2}{(1-\nu_1^2)} [v^o_{,y} + (1/2) w^2_{,y}] + Q_4 \frac{\nu_1 E_1 h_1^2}{(1-\nu_1^2)} [u^o_{,x} + (1/2) w^2_{,x}] - Q_7 \frac{E_1 h_1^3}{12(1-\nu_1^2)} [w_{,yy}] + Q_8 \frac{\nu_1 E_1 h_1^3}{12(1-\nu_1^2)} [w_{,xx}] \qquad (12\text{-e})$$

$$M_{xy} = Q_6 G_1 h_1 [u^o_{,y} + v^o_{,x} + w_{,x} w_{,y}] - 2 Q_9 \frac{G_1 h_1^3}{12} \{w_{,xy}\} \qquad (12\text{-f})$$

Where, Q_1, Q_2, and Q_5 represent the contribution in the membrane rigidities of the plate due to the damping layer. They are calculated and found to be:

$$Q_1 = \left\{ 1 + \frac{E_2 h_2 (1-\nu^2_1)}{E_1 h_1 (1-\nu^2_2)} \right\} \qquad (13\text{-a})$$

$$Q_2 = \left\{ 1 + \frac{\nu_2 E_2 h_2 (1-\nu^2_1)}{\nu_1 E_1 h_1 (1-\nu^2_2)} \right\} \qquad (13\text{-b})$$

$$Q_5 = \left\{ 1 + \frac{G_2 h_2}{G_1 h_1} \right\} \qquad (13\text{-c})$$

Q3, Q4, and Q6 represent the contribution in the coupling rigidities of the plate due to the damping layer. They are also calculated and found to be:

$$Q_3 = \left\{ -\frac{m}{h_1} + \frac{E_2 h_2^2 (1-\nu^2_2)}{E_1 h_1^2 (1-\nu^2_1)} \left(\frac{1}{2} + \frac{h_1}{2h_2} - \frac{m}{h_2} \right) \right\} \qquad (13\text{-d})$$

$$Q_3 = \left\{ -\frac{m}{h_1} + \frac{\nu_2 E_2 h_2^2 (1-\nu^2_1)}{\nu_1 E_1 h_1^2 (1-\nu^2_2)} \left(\frac{1}{2} + \frac{h_1}{2h_2} - \frac{m}{h_2} \right) \right\} \qquad (13\text{-e})$$

$$Q_9 = \left\{ -\frac{m}{h_1} + \frac{G_2 E_2 h_2^2 (1-\nu^2_1)}{G_1 E_1 h_1^2 (1-\nu^2_2)} \left(\frac{1}{2} + \frac{h_1}{2h_2} - \frac{m}{h_2} \right) \right\} \qquad (13\text{-f})$$

Q_7, Q_8, and Q_9 represent the contribution in the flexural rigidities of the plate due to the damping layer. They are calculated and found to be:

$$Q_7=\left\{(1+12\frac{m^2}{h^2_1})+\frac{E_2h_2^3(1-\nu^2_1)}{E_1h_1^3(1-\nu^2_2)}[4+6\frac{h_1}{h_2}+3\frac{h^2_1}{h^2_2}-12\frac{mh_1}{h^2_2}+12\frac{m^2}{h^2_2}-12\frac{m}{h_2}]\right\} \quad (13\text{-g})$$

$$Q_8=\left\{(1+12\frac{m^2}{h^2_1})+\frac{\nu_2E_2h_2^3(1-\nu^2_1)}{\nu_1E_1h_1^3(1-\nu^2_2)}[4+6 \quad + 3\frac{h^2_1}{h^2_2}-12\frac{mh_1}{h^2_2}+12\frac{m^2}{h^2_2}-12\frac{m}{h_2}]\right\} \quad (13\text{-h})$$

$$Q_9=\left\{(1+12\frac{m^2}{h^2_1})+\frac{G_2h_2^3(1-\nu^2_1)}{G_1h_1^3(1-\nu^2_2)}[4+6\frac{h_1}{h_2}+3\frac{h^2_1}{h^2_2}-12\frac{mh_1}{h^2_2}+12\frac{m^2}{h^2_2}-12\frac{m}{h_2}]\right\} \quad (13\text{-i})$$

As shown in Ref. [10] and [15], these resultant stresses and couples are substituted into the equations of motion to get the modified equations with the free layer damping. In fact, q, in equations will stand for the aerodynamic pressure. It is also to be noted that the modulus of elasticity and the shear modulus for the damping layer in the previous equations is given by its complex form as:

$$E_2=E^*_2(1+i\eta_D) \;\&\; G_2=G^*_2(1+i\eta_D) \quad (14)$$

Thus, there are three equations of motion in term of the displacements. These equations again are nondimensionlized and solved by the same procedures given in Ref. [10] and [15]. The effect of the free damping layer is shown in Fig. 4. In this figure it is clear that with the use of damping layer, the critical aerodynamic loading will be shifted to a higher value which means that, at the original given flight regime, there will be no flutter at all.

III.3 Shear Damping Treatment

In this technique the viscoelastic damping layer over the plate is constrained by a third layer called constraining layer, see Fig 5. As the plate vibrates in bending, the constraining layer will constrain the damping layer and force it to deform in shear beside the extension and the compression. This shear motion in the damping layer will result in more dissipation of energy in the system. The ability of the third (constraining) layer to produce a shear motion in the damping layer without itself experiencing excessive stretching is one of the important features of this type of damping. Although the constraining layer must be stiffer than the damping layer, it is not necessarily stiffer than the base plate. In our formulation we consider that the material of this layer is the same as that for the plate. in this case is determined in different manner which is the main difference from the previous one in section **III.2**. The calculation of the neutral plane shift has been done by Ref. [8], [9], [11], [12], [13], and [14]. Here we will present the basic relations and the final expression for m. Fig. 7., shows an element of unit width of the three-layers plate in flexural vibration. The total bending moment M may be expressed as:

$$D\,\frac{\partial\Phi}{\partial x}=\sum_{i=1}^{3}M_{ii}+\sum_{i=1}^{3}N_i\,h_{io} \quad (15)$$

Where :

D - the flexural rigidity of the cross-section.

M_{ii} - the moment exerted by the forces on the i^{th} layer about its own neutral plane.

N_i - the net extensional forces on the i^{th} layer.

h_{io} - the distance from the center of the i^{th} layer to neutral plane of the composite plate.

The various individual moments M_{ii} may be expressed in terms of the curvatures as:

$$M_{11}=\frac{E_1\,h_1^3}{12}\,\frac{\partial\Phi}{\partial x} \quad (16\text{-a})$$

$$M_{22}=\frac{E_2\,h_2^3}{12}\,(\frac{\partial\Phi}{\partial x}-\frac{\partial\psi}{\partial x}) \quad (16\text{-b})$$

$$M_{33}=\frac{E_3\,h_3^3}{12}\,\frac{\partial\Phi}{\partial x} \quad (16\text{-c})$$

The distributions of the extensional strain and stress for the three-layers element of the composite plate are shown in Fig. 8., from which it is possible to deduce that the net extensional force in a given layer is given by the product of the extensional stiffness of the layer and the extensional strain at its mid-plane, i.e.:

$$N_1 = E_1 h_1 h_{1o} \frac{\partial \Phi}{\partial x} \qquad (17\text{-a})$$

$$N_2 = E_2 h_2 \left(h_{2o} \frac{\partial \Phi}{\partial x} - \frac{h_2}{2} \frac{\partial \psi}{\partial x}\right) \qquad (17\text{-b})$$

$$N_3 = E_3 h_3 \left(h_{3o} \frac{\partial \Phi}{\partial x} - h_2 \frac{\partial \psi}{\partial x}\right) \qquad (17\text{-c})$$

where: Φ Is the flexural angle ($\partial w/\partial x$).
ψ Is the shear strain.

In case of pure bending the net extensional force on each composite element must vanish, and by expressing each h_{io} as h_{i1} −m, one may solve for the shift m of the neutral plane by setting the sum of the forces equal to zero, i.e. :

$$\sum_{i=1} N_i = 0 \qquad (18)$$

from which the neutral plane shift m is calculated to be:

$$m = \frac{E_2 h_2 h_{21} + E_3 h_3 h_{31} - [\frac{E_2 h_2}{2} - E_3 h_3] h_2 \frac{\partial \psi}{\partial \Phi}}{E_1 h_1 + E_2 h_2 + E_3 h_3} \qquad (19)$$

To find $\partial\psi/\partial \Phi$, the stress-strain relation for the middle layer may be written as:

$$\psi = -\frac{1}{G_2} \frac{\partial N_3}{\partial x} \qquad (20\text{-a})$$

Also the shear strain, ψ, itself is related to its second derivative by:

$$\psi = -\frac{1}{p^2} \frac{\partial^2 \psi}{\partial x^2} \qquad (20\text{-b})$$

where p is the wave number given by:

$$p = \sqrt[4]{\rho \omega^2 h/D} \qquad (20\text{-c})$$

From (47-a) and (47-b) we find that:

$$h_2 = \frac{\partial \psi}{\partial x} \frac{h_{31} - m}{1 + g} \qquad (21)$$

where the shear parameter g is given by:

$$g = \frac{G_2}{E_3 h_3 h_2 p^2} \qquad (22)$$

Substituting from (20-c) into (22) then the result into (21) the relation between the shear strain ψ and the flexural angle Φ is obtained. The final expression for the neutral plane shift after substituting from (21) into (19) will take the form:

$$m = \frac{g[E_2 h_2 h_{21} + E_3 h_3 h_{31}] - [h_{21} - h_2 E_2 \frac{h_{31}}{2}]}{g[E_1 h_1 + E_2 h_2 + E_3 h_3] + [E_1 h_1 + E \frac{h}{2}]} \qquad (23)$$

Some approximation can be made to find a simplified form of the neutral plane shift. This approximation is based on the assumption that the extensional stiffness of the layer adjoining the base plate is very small compared to that base plate itself. It is also assumed that the stiffness of the constraining layer is at most one-fourth or one-fifth that of the plate to be damped. That is:

$$E_2 h_2 << E_1 h_1 \text{ and } (E_3 h_3)^2 << (E_1 h_1)^2$$

Consequently, with these assumptions in mind the neutral plane shift will be given as:

$$m \approx \frac{g E_3 h_3 h_{31}}{E_1 h_1 + g[E_1 h_1 + E_3 h_3} \qquad (24)$$

The forgoing analysis has only one major restriction, that the three layers of the composite plate must do the same motion. This requires that the wavelength of the flexural vibration be the same in all three layers. Thus, the flexural wave motions in the two outer layers must be closely coupled to the middle layer. If the shear parameter, g, is very small relative to unity, then wave motions can occur in the middle layer itself, so that the two outer layers can become decoupled. However, since we are using thin plate theory in our formulation, it will be assumed that all thicknesses are small relative to the wavelengths of all possible types of wave motion in all layers. Once the neutral plane shift, m, is calculated, then from the strain-displacement and the stress-strain relations, the resultant stresses N_x, N_y, N_{xy} and the resultant couples M_x, M_y, M_{xy} for the composite plate can be derived, and given as:

$$N_x=Q_1^*\frac{E_1 h_1}{1-\nu_1^2}[u^o_{,x}+(1/2)w^2_{,x}]+Q_2^*\frac{\nu_1 E_1 h_1}{1-\nu_1^2}[v^o_{,y}+(1/2)w^2_{,y}]-Q_3^*\frac{E_1 h_1^2}{1-\nu_1^2}[w_{,xx}]-Q_4^*\frac{\nu_1 E_1 h_1^2}{1-\nu_1^2}[w_{,yy}] \quad (25\text{-a})$$

$$N_x=Q_1^*\frac{E_1 h_1}{1-\nu_1^2}[v^o_{,y}+(1/2)w^2_{,y}]+Q_2^*\frac{\nu_1 E_1 h_1}{1-\nu_1^2}[u^o_{,x}+(1/2)w^2_{,x}]-Q_3^*\frac{E_1 h_1^2}{1-\nu_1^2}[w_{,yy}]-Q_4^*\frac{\nu_1 E_1 h_1^2}{1-\nu_1^2}[w_{,xx}] \quad (25\text{-b})$$

$$N_{xy}=Q_5^* G_1 h_1[u^o_{,y}+v^o_{,x}+w_{,x}w_{,y}]-2Q_6^* G_1 h_1^2[w_{,xy}] \quad (25\text{-c})$$

$$M_x=Q_3^*\frac{E_1h_1^2}{1-\nu_1^2}[u^o_{,x}+(1/2)w^2_{,x}]+Q_4^*\frac{\nu_1 E_1 h_1^2}{1-\nu_1^2}[v^o_{,y}+(1/2)w^2_{,y}]-Q_7^*\frac{E_1 h_1^3}{12(1-\nu_1^2)}[w_{,xx}]-Q_8^*\frac{\nu_1 E_1 h_1^3}{12(1-\nu_1^2)}[w_{,yy}] \quad (25\text{-d})$$

$$M_y=Q_3^*\frac{E_1h_1^2}{1-\nu_1^2}[v^o_{,y}+(1/2)w^2_{,y}]+Q_4^*\frac{\nu_1E_1h_1^2}{1-\nu_1^2}[u^o_{,x}+(1/2)w^2_{,x}]-Q_7^*\frac{E_1h_1^3}{12(1-\nu_1^2)}[w_{,yy}]-Q_8^*\frac{\nu_1E_1h_1^3}{12(1-\nu_1^2)}[w_{,xx}] \quad (25\text{-e})$$

$$M_{xy}=Q_6^*G_1h_1^2[u^o_{,y}+v^o_{,x}+w_{,x}w_{,y}]-2Q_9^*[w_{,xy}] \quad (25\text{-f})$$

where Q_1^*, Q_2^*, and Q_5^* represent the contribution in the membrane rigidities of the plate due to the damping and constraining layers. They are calculated and found to be:

$$Q_1^*=[1+\frac{h_3}{h_1}+\frac{E_2 h_2}{E_1 h_1}\frac{1-\nu_1^2}{1-\nu_2^2}] \quad (26\text{-a})$$

$$Q_2^*=[1+\frac{h_3}{h_1}+\frac{\nu_2 E_2 h_2}{\nu_1 E_1 h_1}\frac{1-\nu_1^2}{1-\nu_2^2}] \quad (26\text{-b})$$

$$Q_5^*=[1+\frac{h_3}{h_1}+\frac{G_2 h_2}{G_1 h_1}] \quad (26\text{-c})$$

while Q_3^*, Q_4^*, and Q_6^* represent the contribution in the coupling rigidities of the plate due to the damping and constraining layers. They are also calculated and found to be:

$$Q_3^*=\{-\frac{m}{h}+\frac{h_3^2}{2h_1^2}+\frac{h_2 h_3}{h_1^2}-\frac{m h_3}{h_1^2}+\frac{h_3}{2h_1}+\frac{E_2h_2^2}{E_1h_1^2}\frac{1-\nu_1^2}{1-\nu_2^2}[\frac{1}{2}+\frac{h_1}{2h_2}-\frac{m}{h_2}]\} \quad (26\text{-d})$$

$$Q_4^*=\{-\frac{m}{h_1}+\frac{h_3^2}{2h_1^2}+\frac{h_2 h_3}{h_1^2}-\frac{m h_3}{h_1^2}+\frac{h_3}{2h_1}+\frac{\nu_2E_2 h_2^2}{\nu_1E_1 h_1^2}\frac{1-\nu_1^2}{1-\nu_2^2}[\frac{1}{2}+\frac{h_1}{2h_2}-\frac{m}{h_2}]\} \quad (26\text{-e})$$

$$Q_6^*=\{-\frac{m}{h_1}+\frac{h_3^2}{2h_1^2}+\frac{h_2 h_3}{h_1^2}-\frac{m h_3}{h_1^2}+\frac{h_3}{2h_1}+\frac{G_2 h_2^2}{G_1 h_1^2}[\frac{1}{2}+\frac{h_1}{2h_2}-\frac{m}{h_2}]\} \quad (26\text{-f})$$

and Q_7^*, Q_8^*, and Q_9^* represent the contribution in the flexural rigidities of the plate due to the damping and constraining layers. They are also calculated and found to be:

$$Q_7^*=\{1+12\frac{m^2}{h_1^2}+3\frac{h_3}{h_1}+12\frac{h_2h_3}{h_1^2}-12\frac{mh_3}{h_1^2}+6\frac{h_3^2}{h_1^2}+\frac{m^2h_3}{h_1^3}-24\frac{mh_2h_3}{h_1^3}-12\frac{m h_3^2}{h_1^3}+4\frac{h_3^3}{h_1^3}+12\frac{h_2^2h_3}{h_1^3}+\frac{E_2h_2^3}{E_1h_1^3}\frac{1-\nu_2^2}{1-\nu_1^2}[3\frac{h_1^2}{h_2^2}+6\frac{h_1}{h_2}-12\frac{mh_1}{h_2^2}+12\frac{m^2}{h_2^2}-12\frac{m}{h_2}+12]\} \quad (26\text{-g})$$

$$Q_8^*=\{1+12\frac{m^2}{h_1^2}+3\frac{h_3}{h_1}+12\frac{h_2 h_3}{h_1^2}-12\frac{m h_3}{h_1^2}+6\frac{h_3^2}{h_1^2}+\frac{m^2 h_3}{h_1^3}-24\frac{m h_2 h_3}{h_1^3}-12\frac{m h_3^2}{h_1^3}+4\frac{h_3^3}{h_1^3}+12\frac{h_2^2h_3}{h_1^3}+\frac{\nu_2E_2 h_2^3}{\nu_1E_1 h_1^3}\frac{1-\nu_2^2}{1-\nu_1^2}[3\frac{h_1^2}{h_2^2}+6\frac{h_1}{h_2}-12\frac{m h_1}{h_2^2}+12\frac{m^2}{h_2^2}-12\frac{m}{h_2}+12]\} \quad (26\text{-h})$$

$$Q_9^*=\{1+12\frac{m^2}{h_1^2}+3\frac{h_3}{h_1}+12\frac{h_2 h_3}{h_1^2}-12\frac{m h_3}{h_1^2}+6\frac{h_3^2}{h_1^2}+\frac{m^2 h_3}{h_1^3}-24\frac{m h_2 h_3}{h_1^3}-12\frac{m h_3^2}{h_1^3}+4\frac{h_3^3}{h_1^3}+12\frac{h_2^2h_3}{h_1^3}+\frac{G_2 h_2^3}{G_1 h_1^3}[3\frac{h_1^2}{h_2^2}+6\frac{h_1}{h_2}-12\frac{m h_1}{h_2^2}+12\frac{m^2}{h_2^2}-12\frac{m}{h_2}+12]\} \quad (26\text{-i})$$

Substituting these resultant stresses and couples into the equations of motion, see Ref. [10] and [15], the modified equations with the constrained damping layer are obtained. The modulus of elasticity, and shear modulus of the damping layer are substituted with the complex form given by (14). The three equations obtained are nondimensionalized and again solved by the same procedures shown in Ref. [10] and [15]. The effect of the constrained layer damping on flutter characteristics is shown in Fig. 9. It is clear from the figure that using a constrained layer damping will enable prevention of flutter by shifting the critical aerodynamic pressure at which flutter occurs to a higher value far from that for the given flight regime.

III.4 Comparison between the Two Types of Treatment

It is clear from the previous sections that the two types of damping treatment are able to prevent plate flutter successfully and the question that arises now is, of the two previous techniques, which one is more effective?. To answer this question, a comparison between the two techniques is shown in Fig. 10. It is clear from the figure that for the same weight of plates (which is a very important factor for the design of the aircraft structures), the plate with constrained damping layer is more effective than the plate with free layer damping. However, this efficiency is balanced by greater complication in analysis and application.

Important aspects which must be taken into consideration while choosing the viscoelastic materials are the range of operating temperature and frequency. For the constrained layer damping treatment two extremes must be explained. At low temperatures, where the viscoelastic material is in its glassy region, both the plate and the constrained layer become rigidly coupled and little shear deformation occurs in the middle layer. Hence, the energy dissipation is also small. On the other hand, at high temperature, where the viscoelastic material is in its rubbery region and soft, both the plate and constrained layer become almost uncoupled. The energy dissipation in this case is also minimal, even though the shear deformation in middle layer is high. This is because the shear modulus of the middle layer is low. Between these two extremes, the material possesses an optimal modulus value, so that the energy dissipation will be maximum. The maximum shear deformation in the middle layer is a function of the modulus and thickness of the constraining layer, the thickness of the damping layer, and the wavelength of vibration in addition to the properties of the damping material. The term that contains these variables is the shear parameter, g, given in equation (22). It is to be mentioned that, although the free viscoelastic layer and the constrained viscoelastic layer are useful treatments for damping flexural waves in plates, It is possible to improve their damping performance by introducing a spacer layer between the plate and the treatment see Ref [8]. This ideal spacer layer ideally would have infinite shear stiffness, but an extensional stiffness small relative to that of the plate. The theory for the three-layer plate developed previously in section **III.3**, can be adapted for the case of the spaced constrained viscoelastic layer by assuming that the spacer and the viscoelastic layer act as one composite shear layer. In fact this modified technique increase the loss factor of the system beside that it can help to extent the operating temperature for some higher values. Another modified techniques and materials can also be used in different situations which are not studied here.

IV. Conclusion

Since damping has a great influence on suppressing flutter, one of the methods which is simple and less expensive is to bond a layer of viscoelastic material to the surface of the plate. A modified way which is more efficient is to constrain this layer by a stiffer layer (not stiffer than the plate). This constraining layer operates to shear the damping layer and consequently improve the damping characteristics of the composite plate by dissipating more energy from the system. These two methods depend mainly on the loss factor of the damping layer, which is function of the operating temperature and the frequency range.

The two types of damping treatment are able to prevent plate flutter successfully at its prescribed operating condition. It is clear that for the same weight of plates, the plate with constrained damping layer is more efficient than the plate with free layer damping. However, this efficiency is balanced by greater complication in analysis and application.

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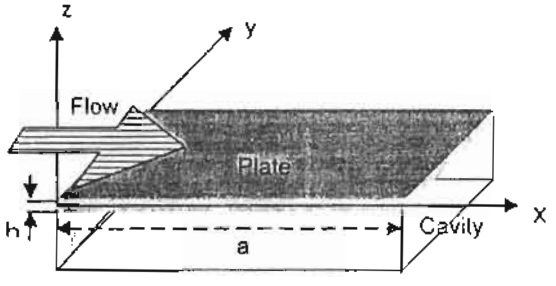

Fig. 1, Geometry of the Plate and Cavity with Axes System

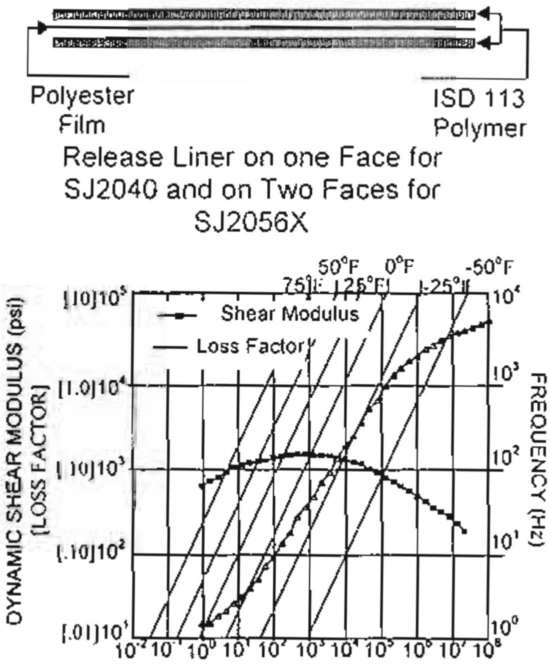

Fig. 2, The Damping Tape Construction and its Characteristic Variations with the Frequency and Temperature, (from 3M Product Information. 3M Industrial

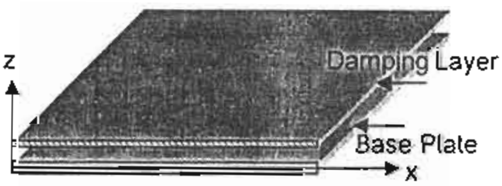

Fig. 3, Plate with Free Layer Damping.

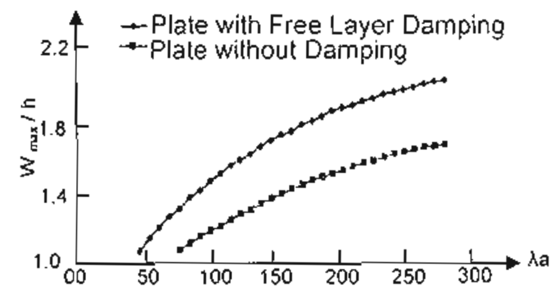

Fig. 4, The Effect of the Free Layer Damping on the Plate Flutter.

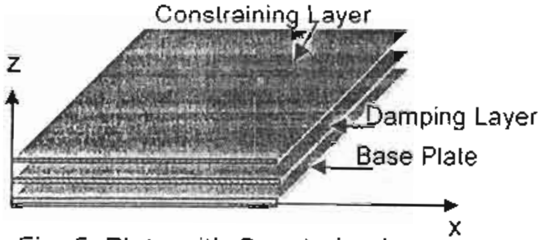

Fig. 5, Plate with Constrained Layer Damping.

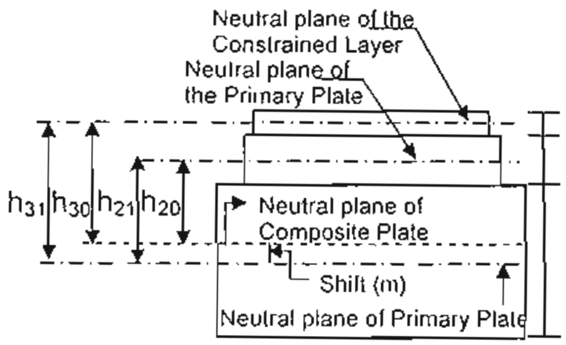

Fig. 6. The neutral plane shift "m" of the Plate with Shear Damping Treatment.

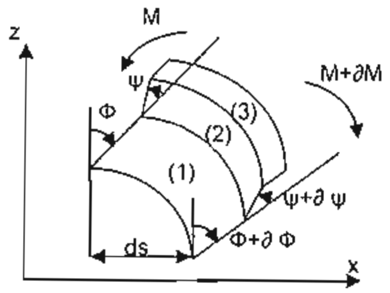

Fig. 7, An Element of the Three-Layer Plate in Flexural Vibration.

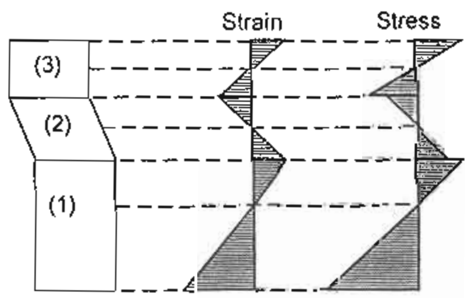

Fig. 8, Extensional Strain and Stress Distributions for the Three-Layer

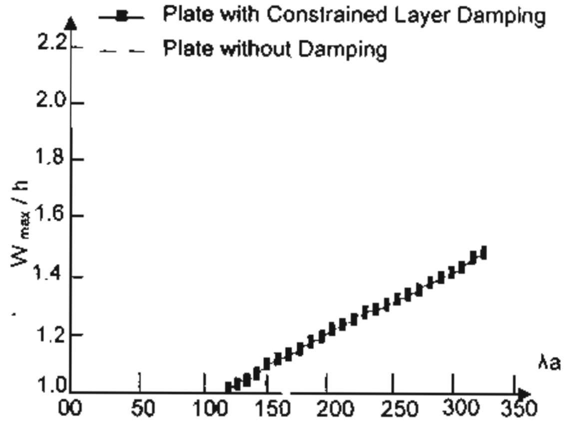

Fig. 9, The Effect of the Constrained Layer Damping on the Plate. Flutter.

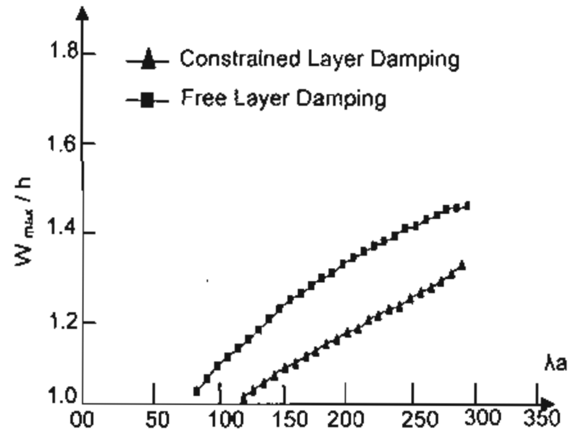

Fig. 10, Comparison by Weight between the Constrained and the Free Layer Damping.

Table 1, Values of α_m and γ_m in the Beam Eigen function or a Clamped Plate.

M	α_m	γ_m
1	4.730040744862704030	0.982502214576223807
2	7.853204624095837557	1.000777311190726905
3	10.99560783800167100	0.999966450125409000
4	14.13716549125746410	1.000001449897656500
5	17.27875965739948100	0.999999937344383000
6	20.42035224562061000	1.000000002707595000
m > 6	(2m+1) π/2	1.0

Table 2, Dimensions and Properties of the Plate and Treatment Damping Layers.

	Plate	Damping Layer	
		SJ2040X	SJ2056X
Availability		Rolls	Sheet
Length (m)	0.7620	27.432	0.9144
Width (m)	0.1778	0.1016	0.5080
Thickness (x 10^{-3} m)	0.8128	0.0215	0.0430
Young's Modulus (x 10^9 N/m^2)	72.398		
Shear Modulus (x 10^9 N/m^2)	27x10^3	19.995	21.995
Poisson's Ratio	0.3300	0.4500	0.4500
Density (x 10^3 kg/m^3)	1.6189	0.9800	0.9800
Coefficient of Thermal Conductivity (μm/m.k)	11.700		
Loss Factor	0.0050	1.0000	1.2000

Table 3, Experimental & Calculated Plate Frequencies for Different Boundary Conditions.

Clamped	Simply Supported	Elastically Supported	Experimental Result
158	72	130.9	128
165	82	138.5	136-142
178	99	160.2	154-161
198	123	181.9	173-180
225	154	200.6	198-206
260	192	216.9	216-228
302	236	261.5	262-275
352	288	308.0	310-324