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TWO DIMENSION VERTICALLY INTEGRATED NUMERICAL SUSPENDED SEDIMENT TRANSPORT MODEL

دراسة عددية بإستخدام نموذج ثنائي الأبعاد لنمذجة حركة الرسوبيات المعلقة

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خلاصة:

يهدف هذا البحث إلي عمل نموذج ثنائي الأبعاد حيث تم أخذ متوسط التركيزات في البعد الرأسي (Z) ودراسة التغيرات في البعدين الأخرين (X,Y) مع التغير في الزمن (t) لمحاكاة حركة الرسوبيات المعلقة مع السريان. حيث يتم فصل معادلة الإنتقال والإنتشار(Advection-Diffusion) إلي معادلتين منفصلتين بإستخدام طريقة الكسور الجزئية : معادلة الانتقال (Advection) ويتم حل هذة المعادلة بطريقة حسابية أما معادلة الإنتشار (Diffusion) يتم حلها بإستخدام (Semi-implicit finite difference).

ABSTRACT:

The objective of this study is to develop a vertically integrated two dimensional numerical sediment transport model. This model is divided in two parts: hydrodynamic modeling and sediment transport modeling. Hydrodynamic modeling simulates flow velocities which are then used in the sediment transport model to simulate sediment concentrations. To represent the sediment transport system in a flow, the conservative form of two dimensional advection diffusion equation is used. To solve this equation a fractional step method, also known as standard split approach (Sobey 1983, Dragsolav 2001), is used. This approach splits the advection diffusion equation in two parts: advection and diffusion, which are solved separately. To solve the advection part, a high resolution conservative algorithm for advection in incompressible flow developed by Leveque (1996) is used. To solve the diffusion part, a semi-implicit finite difference scheme is used.

1-INTRODUCTION:

The oldest known sediment transport study was done around 4000 years ago in China, **Fig. 1.** (Al-Khalif, 1965) A significant work has been done in the last century in the field of sediment transport. All the studies can be classified in two broad categories: physical and mathematical.

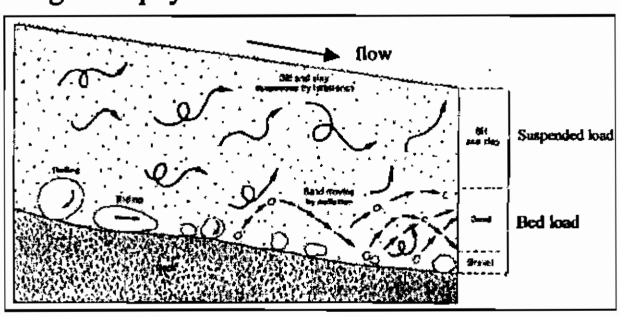

Fig.1: Schematic diagram for different modes of sediment transport (After Al-Khalif, 1965)

Physical studies are done by doing experiments in laboratory flumes or by taking field observations. Laboratory studies are not well representative of the river system as it is difficult to represent a river by a laboratory flume. So a lot of assumptions are usually incorporated in laboratory studies. Still these laboratory studies are important for verification of other studies and also to understand basic concepts of river flow and sediment transport. One of the oldest and still widely used studies was done by Newton (1951). Also, Bhamidipaty (1971) did extensive laboratory flume studies for three different sediment particle sizes using uniform sediment grain size for each experimental run.
Normally, analytical solutions are developed in those cases where flow conditions are very

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simplified and can be lumped in one or two directions. It is difficult to develop analytical solutions for generalized two or three dimensional cases with complex conditions. Still analytical solutions are important to verify the numerical model as it is very difficult to obtain experimental data for many conditions. Some of the well known analytical sediment models are summarized below. One of the oldest models was developed by Tinney (1955). Al-Khalif (1965) used the Einstein (1950) approach to develop a bed load function for a degrading channel and used that function to describe degradation. De Vries (1973) used convection-acceleration and depth gradient terms and developed a linear hyperbolic bed elevation change model. Jaramillo (1983) solved the linear parabolic sediment transport model to estimate bed load discharge for a semi-infinite and finite domain. He estimated the bed elevation using the expression of bed load discharge rate and sediment continuity equation. Gill (1983) developed a model to simulate the bed change in both aggradation and degradation using a linear parabolic bed elevation analytical model for a finite length channel. Jaramillo and Jain (1984) developed a nonlinear parabolic sediment model without considering flow non-uniformities. Jain (1985) used the method of weighted residuals to solve a nonlinear parabolic aggradation model. Zhang and Kahawita (1987) solved a nonlinear parabolic aggradation model and showed that bed elevation is a function of square root of time. Mosconi (1988) developed two different models separately for aggradation and degradation processes. He developed a linear hyperbolic analytical model for aggradation in the case of increase of sediment discharge and nonlinear parabolic analytical model for degradation in the case of reduction of sediment discharge.

All analytical mathematical models of sediment transport phenomena developed are based on the assumption of steady state or quasi steady state water flow, as unsteady state of water flow makes the system complex and it is difficult to develop an analytical solution for that complex system. This assumption is normally not valid in real life problems. To overcome this limitation investigators developed numerical methods to solve sediment transport equations in complex situations. This approach is further encouraged by advancement in the field of computers as these methods need enormous computation. Till now many numerical sediment transport models have been developed. All numerical models developed so far can be divided in three categories according to dimensions in one dimensional, two dimensional and three dimensional models. Some of the widely known and used numerical models are listed below.

In one dimensional sediment transport modeling concentration is averaged in lateral and vertical directions. This is the simplest mode of sediment transport modeling as it involves equations only in one direction. It is easy to implement this approach as analytical solutions can be developed easily for one dimensional differential equations, but this approach cannot be implemented in the case where longitudinal or vertical flow is also important.

In two dimensional sediment transport model sediment concentration is averaged in one direction, normally in vertical direction depending upon the flow characteristics and field requirements. Based on this integration two dimensional models can be classified as depth integrated and laterally integrated two dimensional models. In depth integrated models all thc model parameters and variables are assumed to be the same throughout a water column. Application of two dimensional models is more complicated as compared to one dimensional models as this approach needs more resources in all aspects. Two dimensional models are most popular models than others as they provide enough information of the desired quantity for the project requirement in optimum resources.

Some of the two dimensional models developed so far are described in the next section. Struiksma (1985) developed a two dimensional sediment transport model to simulate the large scale bed change at Delft Hydraulics. Shimizu and Itakura (1989) developed a two dimensional bed load transport model for alluvial channels. Chaudhary (1996) developed a two dimensional bed load sediment transport model for straight and meandering channels. Some of the widely used two dimensional sediment transport models are MIKE21 (DHI 2003), TABS-MD (Thomas and McAnally, 1990), CCHE2D (Wu W., 2001) and HSCTM2D (Hayter, 1995). One of the most popular sediment transport models is CCHE2D sediment transport model (Wu, 2001) developed at the National Center for Computational Hydroscience and Engineering, University of Mississippi. The CCHE2D model has a non equilibrium sediment transport model for suspended load and an equilibrium sediment transport model for bed load. The CCHE2D model is capable of taking account of non-uniform sediment mixtures with many size classes. In the CCHE2D model an exponential difference scheme is used to solve the suspended sediment transport equation and first order upwind scheme is used to solve the bed load transport equation. HSCTM2D (Hydrodynamic, Sediment and Contaminant Transport Model) model was developed for U.S. Environmental Protection Agency. It is a finite element two dimensional, vertically integrated model for cohesive sediments. HSCTM2D is composed of two parts. The first is hydrodynamic modelling part named as HYDRO2D and second is contaminant and sediment transport modelling part known as CS2D. HSCTM2D can be used for both short term and long term simulations.

Three dimensional sediment transport models are most informative as they include all the space dimensions. They are most complicated and resource consuming in implementation. Three dimensional models are avoided until very detailed distribution of desired quantity needs to be simulated and flow characteristics are important in all directions. Three dimensional models are mostly applied in the condition when flow is stratified like flow of fresh water over salt water or flow of warm water over cold water. Many researchers have developed three dimensional models till now. Wang and Adeff (1986) developed a three dimensional finite element model for unsteady flow. Demuren and Rodi (1986) developed a three dimensional flow and neutral tracer transport model. Van Rijn (1987) combined three dimensional sediment transport model and two dimensional depth integrated flow model. Lin and Falconer (1996) developed a three dimensional model for estuaries and coasts.

2- MATHEMATICAL STUDIES

Physical studies have the limitations due to the complexity of representing a real life river conditions through an experimental flume. Due to this restriction, investigators made many assumptions during the experimental runs according to the requirement of the study. These assumptions limited the scope of these studies to apply them to real life problems.

To overcome this problem many investigators developed mathematical equations and their solutions to represent the sediment transport concepts in real life situations. All the mathematical models developed so far are based on the following five basic equations. These equations are written only in one dimension and can be extended for all three dimensions.

(1) Continuity equation for water flow

$$\frac{\partial Q}{\partial x}+\frac{\partial A}{\partial t}=0.0 \qquad (1)$$

where;
Q = discharge
A = cross-section area
x = x-direction, t = time

(2) Momentum equation for water flow

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+gA\frac{\partial z}{\partial x}=0.0 \qquad (2)$$

where;
g = gravitational acceleration
z = flow depth

(3) Flow resistance equation

$$U= a\, S^b \qquad (3)$$

where;
a, b = parameters
S = bed slope

(4) Continuity equation for sediment

$$\frac{\partial A}{\partial t}+\frac{1}{1-\lambda}\frac{\partial G}{\partial x}=0.0 \qquad (4)$$

where;
λ = porosity of sediment mixture
G = sediment transport rate

(5) Sediment transport capacity equation

$G = c\,U^d$ (5)

where;

c, d = parameters

U = mean flow velocity

3- ADVECTION-DIFFUSION EQUATION

The advection diffusion equation could be given

$$\frac{\partial c_k}{\partial t}+\frac{\partial (uc_k)}{\partial x}+\frac{\partial (vc_k)}{\partial y}+\frac{\partial (wc_k)}{\partial z}-\frac{\partial (wsc_k)}{\partial z}= \frac{\partial}{\partial x}\left(k_x\frac{\partial c_k}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial c_k}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z\frac{\partial c_k}{\partial z}\right) \tag{6}$$

In which;

C_k = sediment concentration at any point;
w = flow velocity in z-direction;
w_s = settling velocity;
k_x = diffusion coefficient in x-direction;
k_y = diffusion coefficient in y-direction; and
k_z = diffusion coefficient in z-direction.

Equation **6** can be converted in a pure advection system by neglecting the diffusion part and can be written as:

$$\frac{\partial c_k}{\partial t}+\frac{\partial (uc_k)}{\partial x}+\frac{\partial (vc_k)}{\partial y}+\frac{\partial (wc_k)}{\partial z}-\frac{\partial (wsc_k)}{\partial z}=0.0 \tag{7}$$

Equation **6** can be converted in a pure diffusion system by neglecting the advection part and can be written as:

$$\frac{\partial}{\partial x}\left(k_x\frac{\partial c_k}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial c_k}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z\frac{\partial c_k}{\partial z}\right)=0.0 \tag{8}$$

To convert the three dimensional Equation **6** into a two dimensional depth averaged equation, the depth averaged suspended load concentration is defined as:

$$c_k=\frac{1}{h-\delta}\int_{z_b+\delta}^{z_s} c_k\,dz \tag{9}$$

Integrating the three-dimensional advection-diffusion Equation **6** over the suspended load zone,

$$\int_{z_b+\delta}^{z_s}\frac{\partial c_k}{\partial t}+\int_{z_b+\delta}^{z_s}\frac{\partial (uc_k)}{\partial x}+\int_{z_b+\delta}^{z_s}\frac{\partial (vc_k)}{\partial y}+\int_{z_b+\delta}^{z_s}\frac{\partial (wc_k)}{\partial z} -\int_{z_b+\delta}^{z_s}\frac{\partial (wsc_k)}{\partial z}=\int_{z_b+\delta}^{z_s}\frac{\partial}{\partial x}\left(k_x\frac{\partial c_k}{\partial x}\right)+\int_{z_b+\delta}^{z_s}\frac{\partial}{\partial y}\left(k_y\frac{\partial c_k}{\partial y}\right) +\int_{z_b+\delta}^{z_s}\frac{\partial}{\partial z}\left(k_z\frac{\partial c_k}{\partial z}\right) \tag{10}$$

Integration of Equation **10** over the entire flow depth gives the following depth averaged two dimensional advection-diffusion equation:

$$\frac{\partial}{\partial t}[(h-\delta)c_k]+\frac{\partial}{\partial x}[U(h-\delta)c_k]+\frac{\partial}{\partial y}[V(h-\delta)c_k]+ =\frac{\partial}{\partial x}\left(k_x(h-\delta)\frac{\partial c_k}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y(h-\delta)\frac{\partial c_k}{\partial y}\right) +E_k-D_k \tag{11}$$

where; U and V are the depth averaged flow velocities in the X and Y, directions respectively; and E_k and D_k are erosion and deposition terms in upward and downward directions, respectively, and together known as source-sink term in the advection-diffusion equation. The source-sink term can be calculated as:

$$S_k=E_k-D_k=\alpha w_s(c_k*-c_k) \tag{12}$$

where S_k = the source sink term for specified sediment size, α is the non equilibrium adaptation coefficient, w_s is the sediment particle settling velocity, and c_k* is the depth averaged sediment concentration under equilibrium condition or sediment transport capacity.

As the depth of the bed load zone is small compared to the flow depth δ << *h*, Equation **11** can be simplified as follows:

$$\frac{\partial hc_k}{\partial t}+\frac{\partial (Uhc_k)}{\partial x}+\frac{\partial (Vhc_k)}{\partial y}= \frac{\partial}{\partial x}\left(k_xh\frac{\partial c_k}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_yh\frac{\partial c_k}{\partial y}\right)+E_k-D_k \tag{13}$$

4- FRACTIONAL STEP APPROACH

To solve Equation **13**, a fractional step approach, also known as standard split approach (Sobey 1983), is used. In this approach both advection and diffusion parts of the advection diffusion equation are solved separately at each time step. Using a splitting approach very accurate numerical procedure can be used to solve advection and diffusion separately. A questionable part of this approach is that advection and diffusion parts are solved one after another, which makes them discrete, but in real life they occur simultaneously. This step introduces a splitting error in the solution irrespective of the accuracy of the schemes used to solve the advection and diffusion parts. However the magnitude of error is very less. This approach can be justified on the grounds that better and more accurate methods can be implemented for separate solutions of advection and diffusion parts. The fraction step method procedure is explained below. In general an advection diffusion transport equation can be written as:

$$\frac{\partial c}{\partial t} + L_c(c) - L_d(c) = 0.0 \tag{14}$$

where $L_c(c)$ is the advection part and $L_d(c)$ is the diffusion part including all source-sink terms. Equation **14** can be written using the Taylor series expansion for a nth time step as:

$$\frac{c^{n+1} - c^n}{\Delta t} + L_c(c^n) - L_d(c^n) = \frac{\partial^2 c^n}{\partial t^2}\frac{\Delta t}{2} + \ldots\ldots\ldots\ldots = O(\Delta t) \tag{15}$$

Now introducing the fraction step approach and an intermediate variable c', advection and diffusion parts can be written separately as:

$$\frac{c' - c^n}{\Delta t} + L_c(c^n) = \frac{\partial^2 c^n}{\partial t^2}\frac{\Delta t}{2} + \ldots\ldots\ldots = O(\Delta t) \tag{16}$$

$$\frac{c^{n+1} - c'}{\Delta t} - L_d(c^n) = 0.0 \tag{17}$$

Equation **16** is a pure advection equation and so is Equation **17**. Both of these equations can be solved separately. The fraction-step procedure is independent of the scheme used for advection and diffusion parts. Numerical schemes used to solve for the advection and diffusion parts of the advection diffusion sediment transport equation are described below. Discretization of velocity, water depth and sediment concentration over the space is shown in **Fig. 2.**

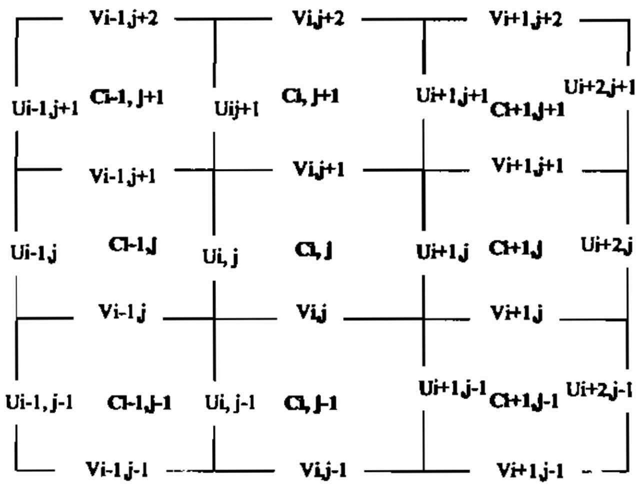

Fig. 2: Discretization of concentration and velocity field in solution domain

5- ADVECTION PART

High resolution conservative algorithm for advection in incompressible flows developed by Leveque (1996) was used for solving the advection part. Leveque uses basic upwind method and proposed several correction terms to achieve better accuracy and stability. A conservative form of advection of a scalar concentration or density function C(x,t) can be written in general as:

$$c_t + \nabla.(\vec{u}.c) = 0.0 \tag{18}$$

Assuming flow is incompressible

$$\nabla\vec{u}(\vec{x}t) = 0.0 \tag{19}$$

From the generalized advection equation, two-dimensional advection equation can be written as:

$$c_t + (cu)_x + (cv)_y = 0.0 \tag{20}$$

and assuming flow is incompressible

$$u_x(x, y, t) + v_y(x, y, t) = 0.0 \text{ for all } x,y,t \qquad (21)$$

For incompressibility in discrete form for every cell in the discretized domain the following condition should satisfy:

$$(U_{i+1,j}{}^{n+1} - U_{i,j}{}^{n+1}) + (V_{i,j+1}{}^{n+1} - V_{i,j}{}^{n+1}) = 0.0 \qquad (22)$$

To solve this conservative form of the advection equation Leveque (1996) used a basic upwind method in the flux differencing and later added correction terms to achieve better accuracy and stability. The upwind method is based on the flux calculation of the concentration at the cell interfaces and can be written as:

$$C_{i,j}{}^{n+1} = C_{i,j}{}^{n+1} - \frac{k}{n}\left[F_{i+1,j} - F_{i,j} + G_{i,j} + 1 - G_{i,j}\right] \qquad (23)$$

where *Fi; j* represents the flux at the left interface of the cell *Ci; j* and *Fi+1; j* represents the flux at the right interface of the cell *Ci; j*. Similarly *Gi; j* represents the flux at the bottom interface of the cell. *Ci; j* and *Gi+1; j* represent the flux at the top interface of the cell *Ci; j*. **Fig. 3** shows the location of flux for a cell.

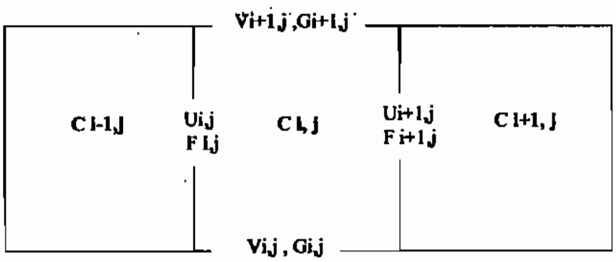

Fig. 3: Representation of flux for a cell

These fluxes at the cell interfaces can be calculated as:

$$Fi.j = v_{i,j}{}^{n+1}.C_{i-1,j}{}^{n}$$

$$Gi,j = v_{i,j}{}^{n+1}.C_{i,j-1}{}^{n} \qquad (24)$$

In this whole section, u and v are taken positive in the X and Y directions, respectively, and all the derivations are done by assuming that u and v are positive. In reality the directions of these fluxes at the interfaces depend upon the direction of the respective velocity vector. Thus Equation **23** can be rewritten as:

$$C_{i,j}{}^{n+1} = C_{i,j}{}^{n+1} - \frac{k}{n}\left[\begin{array}{l} u_{i+1}{}^{n+1}C_{i,j}{}^{n} - u_{i,j}{}^{n+1}C_{i-1,j}{}^{n} \\ + v_{i,j+1}{}^{n+1}C_{i,j}{}^{n} - v_{i,j}{}^{n+1}C_{i,j-1}{}^{n} \end{array}\right] \qquad (25)$$

In this upwind method it is assumed that waves carrying differences (*Ci; j ¡Ci¡1; j*) and (*Ci; j ¡Ci; j¡1*) propagate perpendicular to the interfaces in the X and Y directions, respectively, at the speeds and directions given by velocities u and v. This function can be achieved by using the wave propagation method assuming the above specified condition. In case of wave speed (u, v) in the grid oblique to the interfaces a proper correction factor should be implemented. This correction can be incorporated by a two step procedure. In the first step the same upwind method is used in which wave is propagated perpendicular to the interface and in the next step the remaining triangular part of the wave is used to update the flux between the cells due to its transverse motion. The area of the triangular part of the wave is ***$0.5k^2uv$*** and due to this the cell average is modified by the value of $0.5(k^2/h^2)$ *$uv\Delta c$*. In this quantity *Δc* is the difference across the wave. This modification can be incorporated in the flux calculation of *Fi; j* and *Gi; j* as follows. For wave propagating,

$$\begin{aligned} F_{i,j} &= F_{i,j} + u_{i,j}{}^{n+1}C_{i-1,j}{}^{n} \\ G_{i,j+1} &= G_{i,j+1} - \frac{1}{2}\frac{k}{h}u.v(C_{i,j}{}^{n} - C_{i-1,j}{}^{n}) \\ G_{i,j} &= G_{i,j} + v_{i,j}{}^{n+1}C_{i,j-1}{}^{n} \\ F_{i+1,j} &= F_{i+1,j} - \frac{1}{2}\frac{k}{h}uv(C_{i,j}{}^{n} - C_{i,j-1}{}^{n}) \end{aligned} \qquad (26)$$

The other k/h term is incorporated in the flux differencing expression. This updated form of the upwind method which includes the transverse wave propagation is more stable and accurate than the original version of the upwind method specified in Equation **24**. This improved first order accurate method is known as the coner transport upwind method developed by Collela (1990).

To achieve second order accuracy in the algorithm, a second order Lax-Wendroff method is combined with the upgraded upwind method. The Lax-Wendroff method to calculate flux can be expressed as:

$$F_{i-1,j}^{LW}=\frac{1}{2}u_i(C_{i-1}+C_i)-\frac{k}{2h}u^2(C_i-C_{i-1}) \quad (27)$$

The Lax-Wendroff scheme can also be rearranged as a combination of upwind method and a correction term as:

$$F_{i,j-1}^{LW}=u_iC_{i-1}+\frac{1}{2}|u|\left(1-\frac{k}{h}|u|\right)(C_i-C_{i-1})$$

$$F_{i,j-1}^{LW}=F_{i-1,j}^{UP}+\frac{1}{2}|u|\left(1-\frac{k}{h}|u|\right)(C_i-C_{i-1}) \quad (28)$$

6- DIFFUSION PART

To solve for the diffusion part of the advection diffusion sediment transport equation, a semi-implicit finite difference scheme is used. The semi-implicit finite difference scheme is implemented in such a way that it can easily be converted to a completely explicit or completely implicit scheme. A finite difference representation of the diffusion part can be written as follows:

$$\frac{\partial ch}{\partial t}=\frac{\partial}{\partial x}\left(k_xh\frac{\partial c_k}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_yh\frac{\partial c_k}{\partial y}\right)+S_k \quad (29)$$

where; S_k is the source-sink term, and k_x and k_y are the diffusitivity coefficient in X and Y directions, respectively. Now the above equation can be solved for time steps Δt using an explicit finite difference scheme. In the following solution superscript n represents the nth time step. Introducing a new variable A, Equation **29** can be rewritten as follows:

$$\frac{\partial ch}{\partial t}=\frac{\partial}{\partial x}(A_x)+\frac{\partial}{\partial y}(A_y) \quad (30)$$

where;

$$A_x=K_x.h\frac{\partial c}{\partial x} \quad (31)$$

$$A_y=K_y.h\frac{\partial c}{\partial y} \quad (32)$$

Now, writing the finite difference of Equation **30**

$$\frac{C_{i,j}^{n+1}-C_{i,j}^{n}}{\Delta t}=\frac{Ax(i+1,j)-Ax(i,j)}{\Delta x}+\frac{Ay(i,j+1)-Ay(i,j)}{\Delta y} \quad (33)$$

Here, Ax (i,j) and Ay (i,j) can be calculated as:

$$Ax(i,j)=Kx(i,j).h(i,j)\left[\theta\left(\frac{C^{n+1}(i,j)-C^{n+1}(i-1,j)}{\Delta x}\right)+(1-\theta)\left(\frac{C^{n}(i,j)-C^{n}(i-1,j)}{\Delta x}\right)\right] \quad (34)$$

$$Ay(i,j)=Ky(i,j).h(i,j)\left[\theta\left(\frac{C^{n+1}(i,j)-C^{n+1}(i,j-1)}{\Delta y}\right)+(1-\theta)\left(\frac{C^{n}(i,j)-C^{n}(i,j-1)}{\Delta y}\right)\right] \quad (35)$$

$$Czu2(i,j).C^{n+1}(i+1,j)+Czu1(i,j).C^{n+1}(i-1,j)+Czv2(i,j).C^{n+1}(i,j+1)+Czv1(i,j).C^{n+1}(i,j-1)-C^{n+1}(i,j)=b(i,j) \quad (36)$$

where;

Czu2(i, j) = coefficient of $C_{n+1}(i+1,j)$
Czu1(i, j) = coefficient of $C_{n+1}(i,j-1)$
Czv2(i, j) = coefficient of $C_{n+1}(i,j+1)$
Czv1(i, j) = coefficient of $C_{n+1}(i,j-1)$
b (i, j) = the known terms

Equation **36** can be represented as Ax = b, as it represents a linear system of equations. To solve for the diffusion term, this linear system of equations needs to be solved. To that end, the following numerical schemes was used.

- Jacobi
- Red black gauss siedel
- Successive over relaxation (SOR)

7. SOLVER FOR LINEAR SYSTEMS OF EQUATIONS

- For **Jacobi Method**

A linear system of equations generated from partial differential equations can be solved by using the Jacobi method, which can be written as:

$$u_{i,j}^{n+1} = \frac{1}{4}\left[u_{i-1,j}^{n} + u_{i+1,j}^{n} + u_{i,j-1}^{n} + u_{i,j+1}^{n}\right] \quad (37)$$

where $u^{n}_{i,j}$ denotes the nth iterative value of $u_{i,j}$. Iteration error for the whole grid can be calculated as:

$$Error = \sqrt{\sum (abs(u_{i,j}^{n+1} - u_{i,j}^{n})^2)} \quad (38)$$

- For **Red Black Gauss Seidel Method**

The Red Black Gauss Seidel Method is derived from the Gauss Seidel Method. The Gauss Seidel method iterative formula can be written as:

$$u_{i,j}^{n+1} = \frac{1}{4}\left[u_{i-1,j}^{n+1} + u_{i+1,j}^{n} + u_{i,j-1}^{n+1} + u_{i,j+1}^{n}\right] \quad (39)$$

The difference between Gauss Seidel and Jacobi method is that this method uses the latest iterative values available for the grid points, while the Jacobi method uses only old iterative values for all points. Due to this change, the Gauss Seidel method convergence increases many times more than the Jacobi method. The Red Black Gauss Seidel is a modification of the Gauss Seidel method.

- For **Successive Over-relaxation (SOR) Method**

Successive over-relaxation method can be written as:

$$u_{i,j}^{n+1} = (1-\omega)u_{i,j}^{n} + \frac{1}{4}\omega\begin{bmatrix} u_{i-1,j}^{n+1} + u_{i+1,j}^{n} + u_{i,j-1}^{n+1} \\ + u_{i,j+1}^{n} \end{bmatrix} \quad (40)$$

The rate of convergence of the SOR iteration method depends upon the choice of ω, which is called as accelerating factor and lies between 1 and 2. There is no way to estimate the value of ω for an iteration process for a particular problem. The only way to estimate the value of ω is by hit and trial method. Initially some value of ω is assumed and then it is changed until the best converging rate is achieved. This method is also included in the model. This method is not very good as each time one has to estimate the value of ω for best results. Iteration error for this method can be calculated in the same way as explained in the Jacobi method.

8. RESULTS AND DISCUSSION

- Testing of Advection Algorithm

Advection algorithm described in previous section was tested for a test problem shown in Leveque (1996), who developed the advection algorithm. In the test problem a plane of dimensions 1 x 1 with grid sizes of 100 x 100, which make grid dimensions of 0.01 x 0.01. In this plane a value of density function was assigned, which is shown in the **Fig. 4.** This method is called solid body rotation test. Now a non-constant velocity profile is specified in the plane as:

$$u=-(y-1/2), \quad v=(x-1/2) \quad (41)$$

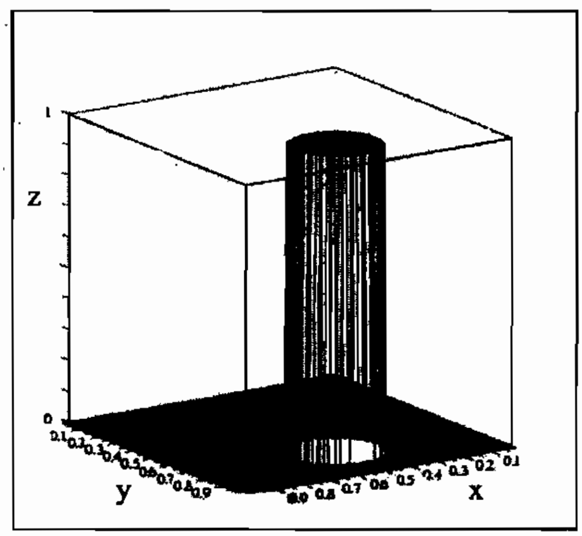

Fig. (4): Initial density function on a plane for solid body rotation test of advection

Initial data of density function in the form of disk is centered at $x_0 = 0.5$ and $y_0 = 0.75$ with a radius of 0.15. A time step value for advection iteration was chosen 0.01. In this test problem pure advection is assumed and the velocity profile is taken in such a way that disk should come back at its original position with the same density function values as at points. This problem was tested by method three with all limiters. The result after one revolution using the third method is shown in **Fig. 5.**

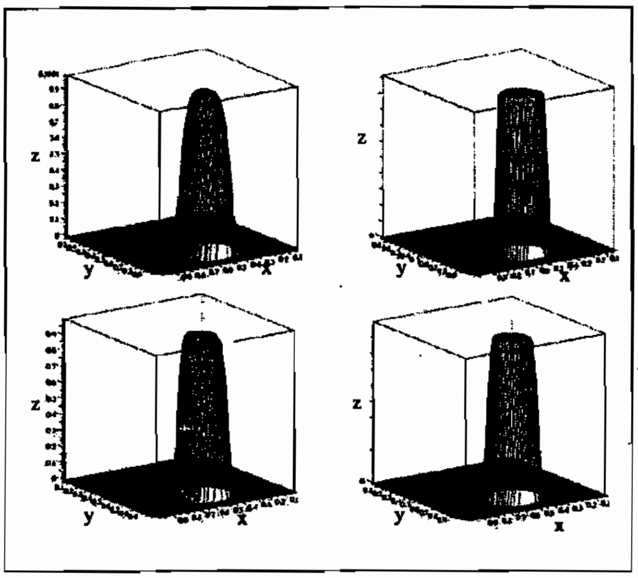

Fig. (5): Profile of density function after one revolution using 3rd method

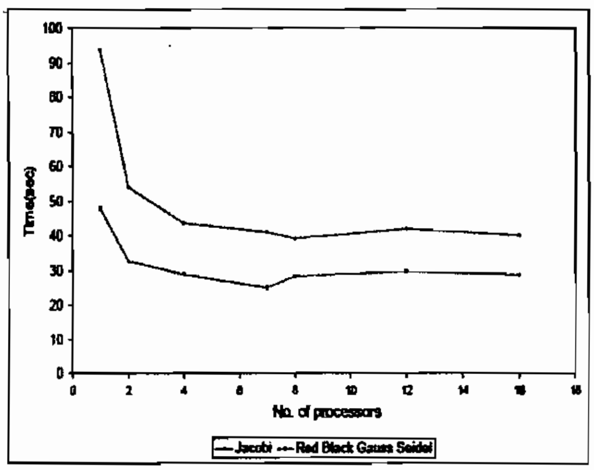

Fig. (6): Scalability and comparison of speed for Jacobi and Red Black Gauss Seidel methods

- Advection-diffusion Combined Test

After testing advection and diffusion schemes separately, the combined advection-diffusion scheme was tested. The combined advection - diffusion scheme was tested for Wexler (1992) analytical solution of two dimensional advection-diffusion including source-sink term and Zoppou's (1997) analytical solution without source-sink term.

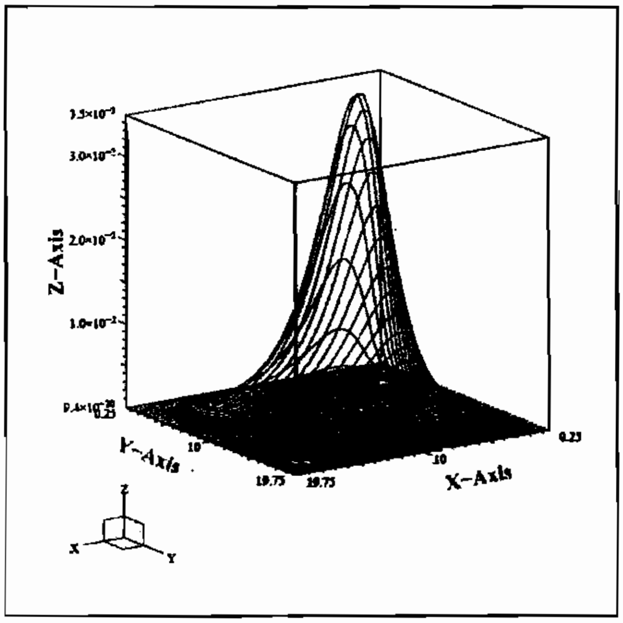

Fig. (7): 3D plot of concentration profile using analytical method at 0.05 second

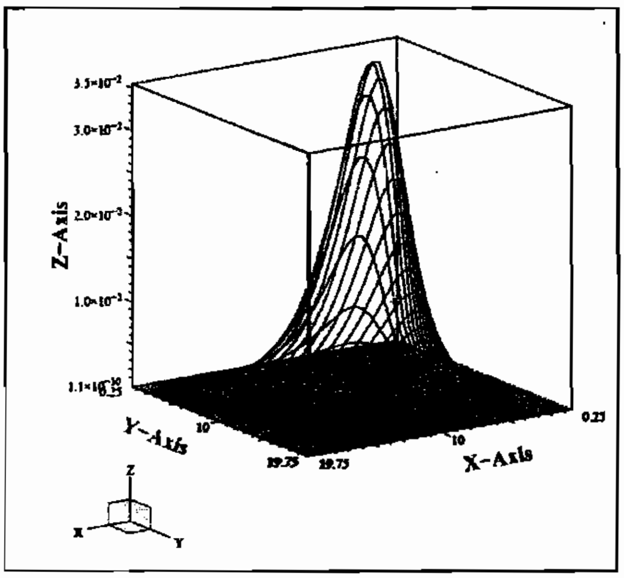

Fig. (8): 3D plot of concentration profile using advection-diffusion numerical scheme at 0.05 second

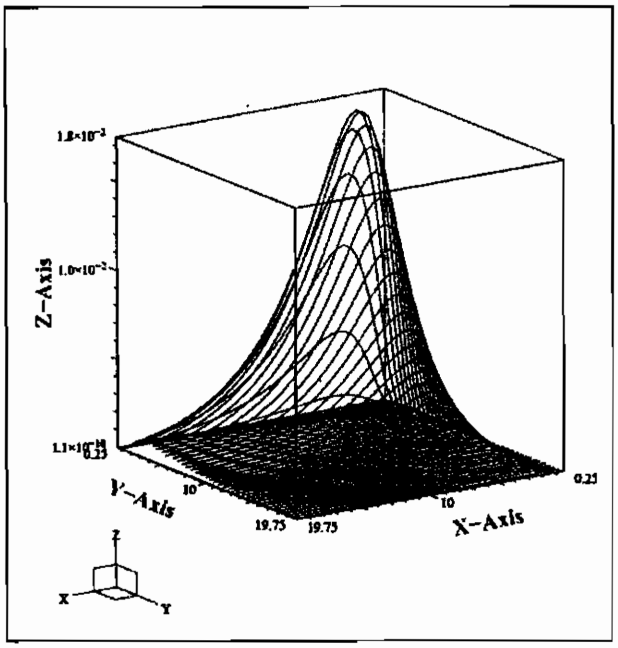

Fig. (9): 3D plot of concentration profile using analytical method at 0.1 second

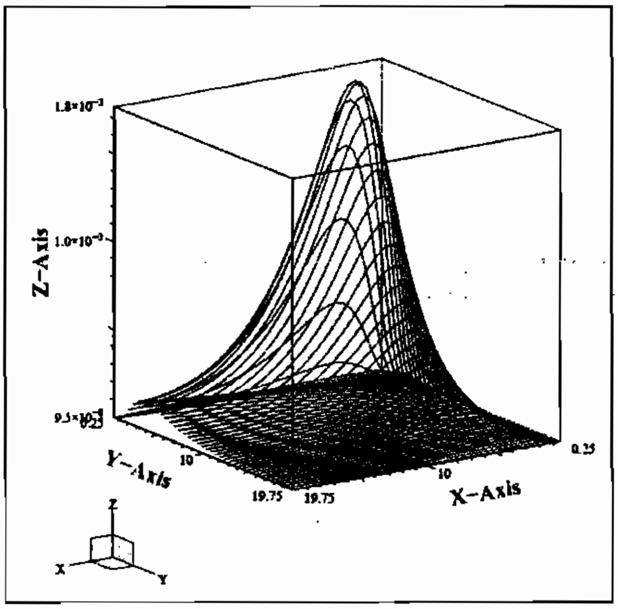

Fig. (10): 3D plot of concentration profile using advection-diffusion numerical scheme at 0.1 second

9. CONCENTRATION PROFILE CONTOUR

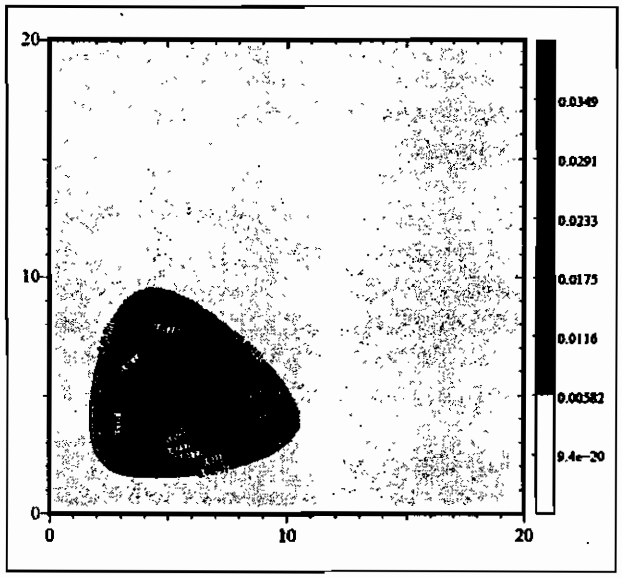

Fig. (11): Concentration profile contour using analytical method and numerical scheme at 0.05 second Filled contour - Analytical method Line contour - Numerical scheme

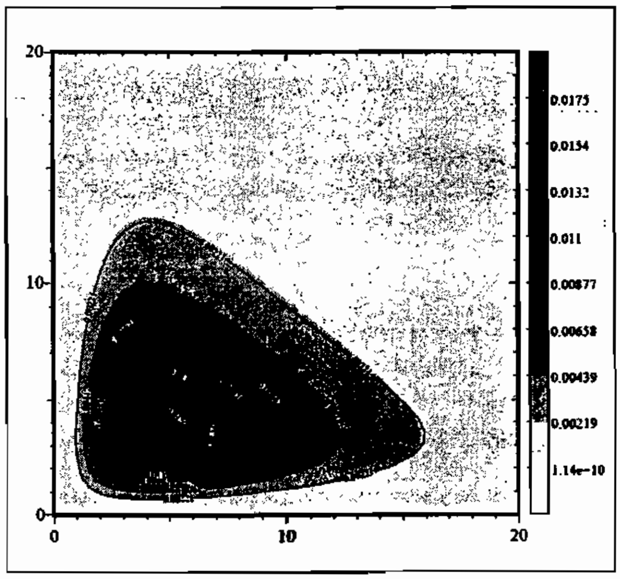

Fig. (12): Concentration profile contour using analytical method and numerical scheme at 0.1 second Filled contour - Analytical method Line contour - Numerical scheme

10. CONCLUSION

- A vertically integrated two dimensional numerical sediment transport model was used to simulate suspended sediment transport using advection-diffusion equation and solved it separately.
- This model is divided in two parts: hydrodynamic modeling and sediment transport modeling. Hydrodynamic modeling simulates flow velocities which are then used in the sediment transport model to simulate sediment concentrations.
- There was a very small difference of 3rd plot profiles of concentration between the advection-diffusion numerical scheme and the analytical method for small time values and *vice versa*.

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