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Transmission Network Cost Allocation Based on Superposition Theorem

توزيع تكاليف النقل باستخدام نظرية التراكب

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ملخص البحث:

لقد أدى الاتجاه المتزايد الي هيكلة خدمات النقل ان أصبح توزيع التكلفة أمر حاسم وهام وصارمن الضروري في نظم القوي المهيكلة تحديد نسب مشاركة الاحمال و المولدات في القدرة المتدفقة عبر خطوط النقل لضمان توزيع عادل للتكلفة بين كل الاطراف المعنية. ويقدم هذا البحث منهجية جديدة لتوزيع تكلفة شبكة النقل على مستخدميها استنادا إلى نظرية التراكب . ومن أهم مميزات هذه الطريقة المقترحة أنها تعتمد على السريان الحقيقي فى الخطوط وتستوعب انعكاس السريان في خطوط النقل بالاضافة الي ان التكلفة الموزعة علي الاحمال والمولدات دائما موجبة وأنها لا تتطلب لتنفيذها سوى معرفة بسيطة بموضوع تدفق القدرة ونظريات الدوائر، وهى لا تعتمد على اختيار Slack bus، كما أنها واضحة فى التطبيق .وقد تم التأكيد على فعالية الطريقة المقترحة بتطبيقها علي منظومتى اختبار قياستين هما IEEE 24 bus RTS وكذلك 4-bus system. وقد تم مقارنة توزيع التكاليف بهذه الطريقة مع التقنيات الأخرى المستخدمة حاليا لتوزيع هذه التكاليف .

Abstract—**Due to continuing trend towards deregulation and unbundling of transmission services, usage cost allocation has become critical. In a deregulated power system, it is usually required to determine the shares of each load and generation in line flows, to permit fair allocation of transmission costs between the interested parties. This paper presents a new methodology for allocating the cost of a transmission network to its users based on the superposition theorem. The suggested methodology has several desirable properties as it is flow-based transmission cost allocation (TCA) method, requires only a simple knowledge of load flow and circuit theorems for its implementation; it is independent on the choice of the slack bus; it is straightforward to apply; depends on the real flow; it is a physical- based technique; recognizes of counter-flows and transmission use charges that are stable and always positive. Effectiveness of the proposed method has been established using both 4-bus system and IEEE 24 bus system and the results are compared with other available TCA techniques (Pro-rata, proportional sharing and equivalent bilateral exchange techniques).**

Index Terms— DC load flow, Superposition theorem, Transmission cost allocation,

1. INTRODUCTION

DIFFERENT proposals for allocating the cost of a transmission and distribution networks have appeared in the last years. Clearly, the focus has been mainly on transmission systems, due to the large number of agents with open access to transmission networks. In this way, several proposals have appeared in the technical literature [1, 6] and some comparative studies were presented. Cost allocation methods are aimed at assigning the responsibility of paying the transmission system total Cost or reinforcement cost to each generator and each load. The suitable TCA method should reflect the power injection at each bus and consider relative position of the bus in the transmission network. It should be consistent and stable with respect to small alternatives of network parameters. Also, it has to provide effective incentives or disincentives for generators and loads regarding their relative locations and finally it is desirable to be simple to understand and implement.

Several TCA methods have been proposed in the last few years. There are methods based on the pro-rata (PR) technique [1], methods based on proportional sharing (PS) [2, 3], methods based on bilateral contracts (EBE) [4] and methods based on physical technique [5]. The Methods based on the pro-rata technique [1] have the advantage that they are simple to calculate and implement. However, it can

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be argued that both generators and loads are charged a flat rate per megawatt-hour, disregarding to their respective use of individual transmission lines.

Other more elaborated methods are flow-based methods as in [6]. These methods estimate the usage of the lines by generators and demands and charge them accordingly. Some flow- based methods use the proportional sharing principle [3]. This method implies that any active power flow leaving a bus is proportionally made up of the flows entering that bus; such that Kirchhoff's current law is satisfied. The methods, that use generation shift distribution factors [7], are dependent on the selection of the slack bus and lead to controversial results. The usage-based method reported in [4] uses the so-called Equivalent Bilateral Exchanges (EBE). To build the EBE, each demand is proportionally assigned a fraction of each generation. Conversely, each generation is proportionally assigned a fraction of each demand, in such a way that both Kirchhoff's laws are satisfied. These techniques require stronger assumptions (the proportional sharing principle, the principle of EBE cannot be proved or disproved), which diminish their practical interest.

This paper presents a new method of allocating the cost of a transmission network based on the superposition theorem. It should be noted that the proposed methodology simply relies on circuit laws while the proportional sharing and EBE technique relies on the proportional sharing principle. That's not needed by the transmission cost allocation based on superposition theorem (SP). Transmission use rates derived from the superposition principle are characterized as simple methodology as it is easy to understand, implement, and calculate. It is based on a famous theory in electrical circuits not based on assumptions (EBE or PS) that can be neither proved nor disproved and finally it is location- dependent rates.

The proposed TCA based on superposition theorem is implemented and compared to other existing TCA methods via a number of simulations on the 4-bus system and on the IEEE 24-bus RTS system.

2. Mathematical Formulation

Superposition theorem is one of those strokes of genius that takes a complex subject and simplifies it in a way that makes common sense. The principle of this theorem states that a circuit can be analyzed with only one source of power at a time. The corresponding component voltages and currents algebraically added to find out what they will do with all power sources in effect. Knowing this allows one to consider complicated circuits in terms of simpler components and then combining the results.

The proposed Transmission cost allocation method uses the superposition theorem as a reference in allocating the cost among the generators and the demands, separately.

2.1. Transmission Cost Allocation to Generators

The algorithm of transmission cost allocation to generators based on the superposition theorem is listed in the following steps:

Step (1): let one source supplies the system demand after reducing the system demand to be suitable to the value of the active generator, the new demand is calculated as shown in Equations 1 and 2.

$$Pd_k^i = \frac{Pd_k \times Pg_i}{total\ demand} \quad (1)$$

$$\text{and } \sum_{k=1}^{K} Pd_k^i = Pg_i \quad (2)$$

Step (2): Run DC load flow to obtain the new flow in lines as in Equation 3.

$$P_m^i = x_m^{-1}\theta_m^i$$

$$\text{and } \theta_m = \theta_{ik} = \theta_i - \theta_k \quad (3)$$

Check if the generator bus number i equals I go to step (3), otherwise, go to step (1) for the next generator bus.

Step (3): After applying the superposition theorem on the last generator, get the total use of line m due to generation cost allocation as in Equation 4.

$$|U_m^g| = \sum_{i=1}^{I} |P_m^i| \quad (4)$$

Using the absolute value here is to charge for the flow irrespective to its direction. In other words, the flow and counter flow are charged for line use indistinguishably. This property is crucial for the stability of rates of use by loads and generators, particularly if some line flows are near zero and can change direction following a small change in the operating conditions.

Step (4): Calculate the total usage of the network by generator as in Equation 5.

$$Ug_i^m = \frac{|P_m^i|}{|U_m^g|} \quad (5)$$

Step (5): If the line cost C_m in U.S.\$/h, then.$TUCG_i$ for the use of all lines, according to the assumption of a 50/50 split in transmission costs between generation and demand is calculated as in Equation 6.

$$TUCG_i = \frac{\sum_{m=1}^{M} C_m Ug_i^m}{2} \quad (6)$$

Where:

k : load bus number = 1..... K,

K : Total number of load buses,

i : Generator bus number= 1... I,

I : Total number of generator buses,

Pd_k : Actual demand at load bus k, MW,

Pd_k^i: Demand at load bus k when loads supplied only from generator i, MW and

Pg_i: Actual generation at generator bus i, MW.
P_m^i : Line flow in line m when loads supplied only from generator i, MW ,
X_m: Series reactance of the branch m,
m: Line number = 1 ...... M,
M: total number of lines,
θ_i, θ_k: voltage angles at the terminal buses of branch m.
U_m^g : total usage of line m due to generator cost allocation.
Ug_i^m: use of line m by generator i as a percentage of the total usage of line m .
C_m : cost of line m in U.S.$/h ,
$TUCG_i$: Transmission charge applied to generator, i .

2.2 Transmission Cost Allocation to Demands

The algorithm of transmission cost allocation to demands based on the superposition theorem is listed in the following steps:

Step (1): let all sources supply one demand at a time after reducing the system generation to match the requirement of that demand, the new generation is calculated as in Equations 7 and 8.

$$Pg_i^k = \frac{Pg_i \times Pd_k}{total\ generation} \quad (7)$$

$$\text{and} \quad \sum_{i=1}^{I} Pg_i^k = Pd_k \quad (8)$$

Step (2): Run DC load flow to obtain new flow in lines using Equation 9.

$$P_m^k = x_m^{-1}\theta_m^k \quad (9)$$

Where $\theta_m = \theta_{ik} = \theta_i - \theta_k$

Check if the load bus number k equals K , go to step (3), otherwise go to step (1) for the following load bus.

Step (3): After applying the superposition theorem on the last demand, get the total use of line m due to demand cost allocation as in Equation 10.

$$|U_m^d| = \sum_{k=1}^{K} |P_m^k| \quad (10)$$

The proposed algorithm treats the flow and the counter flow equally. As, both being assigned a line-use debit, may be contentious in systems where counter flow is deemed to help reduce the use of a line and is, therefore, assigned a line-use credit rather than a debit. It is argued, however, that this distinction between flow and counter flow has a more solid justification when dealing with transmission-loss allocation, where the net flow is crucial in defining the line losses. In transmission-use allocation, however, even in a line with zero flow where counter flows and flows cancel out, the generating and consuming agents must be assigned some transmission use cost for this line.

Step (4): Calculate the total usage of the network by each demand, Ud_k^m using Equation 11.

$$Ud_k^m = \frac{|P_m^k|}{|U_m^d|} \quad (11)$$

Step (5): If the line cost, C_m in U.S.$/h , then the total cost allocated to demand Pd_k for the use of all lines, according to the assumption of a 50/50 split in transmission costs between generation and demand is calculated as in Equation 12

$$TUCD_k = \frac{\sum_{m=1}^{M} C_m Ud_k^m}{2} \quad (12)$$

Where:
k : Load bus number = 1..... K,
K: Total number of load buses) ,
i : Generator bus number = 1... I,
I : Total number of generator buses) ,
Pd_k : Actual demand at load bus k, MW,
Pg_i^k: Generation at generator bus i when generators supply only demand at bus k, MW and
Pg_i: Actual generation at generator bus i, MW
P_m^k : Line flow in line m when generators supply only demand at bus k,MW ,
X_m: Series reactance of the branch, m
m: Line number = 1 ...... M
M: total number of lines),
θ_i, θ_k: Phase angles at the terminal buses of branch m.
U_m^k : Total usage of line m due to demand cost allocation.
Ud_k^m: Use of line m by demand k as a percentage of the total usage of line m .
C_m : cost of line m in U.S.$/h ,
$TUCD_k$: Transmission charge applied to demand, k.

3. Implementation of the TCA Based on Superposition

To make the proposed algorithm clear, the flow diagram of the process of allocating cost to generators is shown in Figure 1. In similar way, the flow diagram of allocating cost to demand is shown in Figure 2.

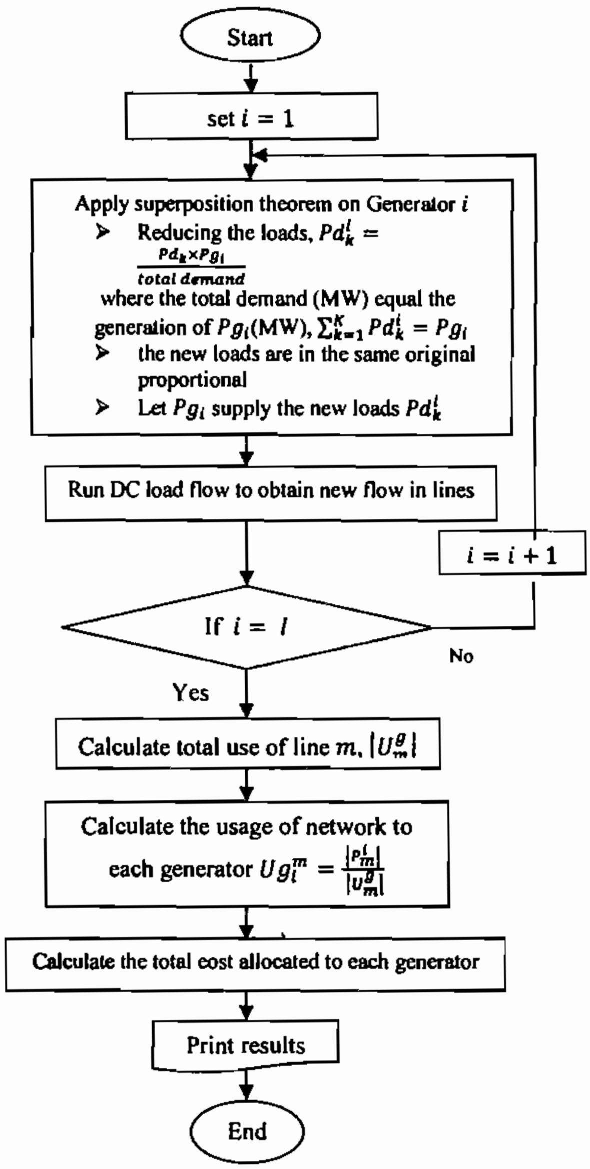

Figure 1: Flow Chart of the Proposed Method that Allocate Cost to Generators

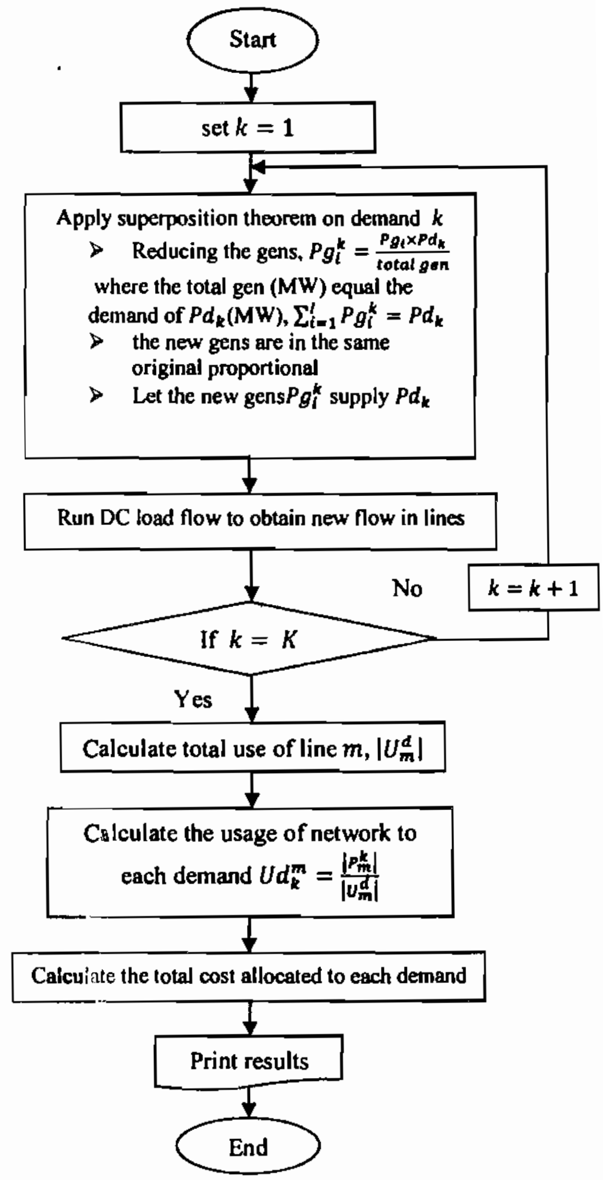

Figure 2. Flow Chart of the Proposed Method that Allocate Cost to Demands

4. Case Studies

The proposed transmission line cost allocation based on superposition is tested for many systems. Here, two of the tested systems are listed namely; 4-bus test system and IEEE 24- bus test system.

4.1. Case 1: 4-Bus Test System

To explain the use of the Superposition theorem in transmission cost allocation, a simple 4-bus test system is considered. Figure 3 shows the system including loads, generations, and line flows, all in megawatt [4]. The line costs in U.S. $/h are shown in Table (1). MATPOWER package version 2.0 and MATLAB 6.5 Software [8,9] are used to Run the DC load flow. Data for buses are given in Table (2). Table (3) shows the system line data.

Table 1: Line resistance and reactance

Line	1	2	3	4	5
$R_m = C_m$	12.75	6	11.7	3.5	5.75
X_m	97	69.5	96	30.8	58

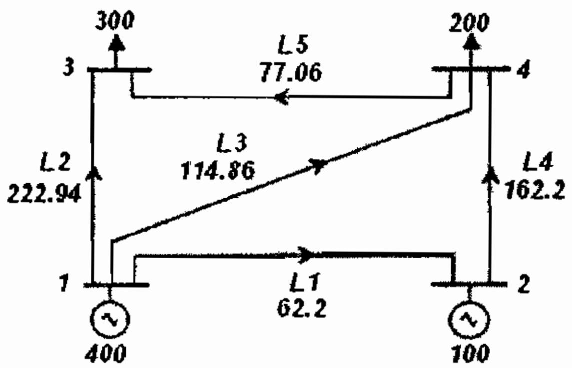

Fig. 3.Real Power Flows in the Four-Bus Test System

Table 2: Bus Data of 4- Bus Test System

Bus	Bus type	[illegible]	[illegible]
1	3	0	400
2	2	0	100
3	1	300	0
4	1	200	0

Table 3: Line Data of 4-Bus Test System

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1	1	2	0.0241	0.1834
2	1	3	0.0113	0.1314
3	1	4	0.0221	0.1815
4	2	4	0.0066	0.0582
5	3	4	0.0108	0.1096

4.1.1 Charges to Generators

Step (1): By applying superposition theorem on the four-bus test system, let generator Pg_1 alone supplies the system demand at buses (3) and (4) after reducing the system demand, the new demand is calculated using equation (1) as follows: $Pd_3^1 = 240\ MW$ and $Pd_4^1 = 160\ MW$.Equation (2) satisfies the constraint:
$\sum_{k=1}^{4} Pd_k^1 = Pd_1^1 + Pd_2^1 + Pd_3^1 + Pd_4^1 = 400$ (the only generation in the network and there is no load at buses 1,2). Note that, the new demand is in the same proportional with the old demand.

Step (2): Run DC load flow using MATPOWER package version 2.0 and MATLAB 6.5 Software and using equation (7), the new flow in lines is obtained, Table 4 presents the new flow in lines.

Table 4: New Line Flow When Pg_1 Supplies System Demand

Branch	[illegible]	[illegible]	[illegible]
1	1	2	87.25
2	1	3	196.61
3	1	4	116.14
4	2	4	87.25
5	3	4	-43.39

Repeat step (1): For generator Pg_2 alone to supply the system demand at buses (3)&(4) after reducing the system demand ,the new demand can be obtained from equation (1) as following $Pd_3^2 = 60\ MW$ and $Pd_4^2 = 40\ MW$ while applying equation (2) to satisfy the power balance.

Repeat step (2): Running the DC load flow, the new flow in lines are obtained and listed in Table 5.

Table 5: New Line Flows When Pg_2 Supplies System Demand

Branch	[illegible]	[illegible]	[illegible]
1	1	2	-25.05
2	1	3	26.32
3	1	4	-1.28
4	2	4	74.95
5	3	4	-33.68

Step (3): Calculate total use of lines using equation 8 as shown in Table 6.

Table 6: Total Use of Lines When Applying Superposition on Generators

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
$\lvert U_m^g \rvert$ (MW)	112.3	222.93	117.42	162.2	77.07

Step(4): The total usage of the network by each generator is obtained from equation 9 as shown in Table 7.

Table 7: Total Usage of the Network by Each Generator

[illegible]	[illegible]	[illegible]	[illegible]
1	77.69	22.31	Line 1 is a direct line between Pg_1&Pg_2& since $Pg_1 > Pg_2$ so, $Ug_1^1 > Ug_2^1$
2	88.19	11.81	Line 2 is a direct line between Pg_1(max gen)& Pd_1(Max demand) So,$Ug_1^2 > Ug_2^2$
3	98.91	1.09	Line 3 is a direct line between Pg_1, Pd_4&Pg_2 has a direct line (Line 4)with Pd_4& since $Pg_1 > Pg_2$ so, $Ug_1^3 > Ug_2^3$
4	53.79	46.21	Since Pg_1 uses line1 more than Pg_2&Pg_2 has a direct line (Line 4)with Pd_4 so,Pg_1&Pg_2 contribute in supplying of Pd_3& Pd_4
5	56.30	43.70	Pg_1& Pg_2contribute in suppling of Pd_3& Pd_4

Step (5): From Equation (6) the total usage cost allocated to generation Pg_i for the use of all lines is obtained after 50/50 split assumption between generation and demand as shown in Table 8.

Table 8: Total Usage Cost Allocated to generation

[illegible]	[illegible]	[illegible]	[illegible]
$TUCG_i$ ($/h)	15.94	3.91	19.85

4.1.2. Charges to Demands

Step (1): let all sources of power Pg_1 & Pg_2 supplies one demand Pd_3 at a time after reducing the system generation at buses (1) &(2) ,the new generation can be obtained from equation (7) as follows:
$Pg_1^3 = 240\ MW$ and $Pg_2^3 = 60\ MW$ that satisfies equation (8).

Step (2): Run DC load flow to obtain the new flow in the lines that's shown in Table (9).

Table 9: Line Flow in Lines When Pg_1^3 & Pg_2^3 Supplies Only Load Pd_3

[illegible]	from bus	to bus	[illegible]
1	1	2	20.95
2	1	3	171.92
3	1	4	47.13
4	2	4	80 95
5	3	4	-128.08

Repeat step(1): let all sources of power Pg_1 & Pg_2 supplies one demand Pd_4 after reducing the system generation at buses (1) &(2). The new generation can be obtained from equation (7) as follows:
$Pg_1^4 = 160\ MW$ and $Pg_2^4 = 40\ MW$ while satisfying Equation (13).

Repeat Step (2): Run DC load flow to obtain the new flow in lines which is listed in Table (10).

Table 10: New Line Flow When Pg_1^4 & Pg_2^4 Supplies Pd_4 only

[illegible]	[illegible]	[illegible]	[illegible]
1	1	2	41.25
2	1	3	51.01
3	1	4	67.74
4	2	4	81.25
5	3	4	51.01

Step (3): The total use of lines can be obtained from Equation (10), Table (11) presents the results.

Table 11: Total Use of Lines

	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
$\|U_m^d\|$ (MW)	62.2	222.93	114.87	162.2	179.09

Step (4): The total usage of the network by each demand can be obtained from equation (11). Table (12) presents the results.

Table 12: Total Usage of Network by Each Demand

[illegible]	[illegible]	[illegible]	Comment
1	33 68	66.32	Demand 3 uses line 2 >line 1 because D3 has a direct line with Pg_1(Max generation)
2	77.12	22.88	$Ud_1^2 \approx Ud_1^5$ because no demand at bus 3 when Pg_1^4 & Pg_2^4 supplies one demand Pd_4
3	41.03	58.97	Pg_1& Pg_2contribute in suppling of Pd_3& Pd_4
4	49.91	50.09	Pg_1& Pg_2contribute in suppling of Pd_3& Pd_4
5	71.52	28.48	$Ud_2^2 \approx Ud_2^5$ because no demand at bus 4 when Pg_1^3 & Pg_2^3 supplies one demand Pd_3

Step (5): From equation (12) the total usage cost allocated to demand Pd_k according to the 50/50 split assumption between generation and demand. Table (13) presents the result.

Table 13: Total Cost Allocated to Demand, Pd_k

[illegible]	[illegible]	[illegible]	[illegible]
$TUCD_k$ ($/h)	9.79	10.06	19.85

The results are shown in Table (14). Adding up the total payments from Table (13) results in U.S.$ 39.7/h, which is the total network cost.

Table 14: Charges for Use of the Network for 4-Bus Test System

[illegible]	Bus				[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]	
TUCG ($/h)	15.94	3.91	-	-	19.85
TUCD ($/h)	-	-	9.79	10.06	19.85

4.1.3. Comparison between the Proposed SP Method and Existing Methods

For validation Purposes, the transmission cost allocated for each load and generator from our proposed superposition method is compared with the corresponding ones. Here, the existing methods are equivalent bilateral exchange (EBE), proportional sharing (PS), and pro rata (PR). The results of the previous three methods can be found in [5]. This comparison is shown in Table 15.

Table 15: Comparison between Different Methods of TCA for 4-BusTest System

[illegible]	[illegible]			
-	SP	EBE[5]	PS[5]	PR[5]
1	15.94	16.21	13.8	15.88
2	3.91	3.64	6.05	3.97
3	9.79	11.19	9.08	11.91
4	10.06	8.66	10.77	7.94
Total	39.7	39.7	39.7	39.7

4.2. Result Analysis

Observing Table (15), it can be noted that, all methods allocate most of the transmission cost to generator at bus1. This is expected because the generator at bus1 is electrically close to the high cost lines 1, 2 and 3, and generator at bus 1 is close to load center. Also, it shows that the SP and PS methods allocate most of the transmission cost to demand at bus 4. On the other hand, the EBE and the PR methods allocate most of the transmission cost to demand at bus 3 although the demand at bus 3 is comparatively high. This is because that demand at bus 4 is directly connected to lines 3, 4 and 5. Those have cost more than lines 2and 5 which are directly connected to demand at bus 3. Additionally, demand at bus 3 is electrically close to heavy generation at bus 1by line 2 that has the highest flow in the network. So, demand at bus 4 use network more than demand at bus 3.

The above results illustrate that the adequacy of the TCA based on SP compared to other methods and show its appropriate behavior.

4.3. Case 2: IEEE 24-Bus RTS System

The transmission network cost allocation based on superposition theorem (SP) method is compared with three alternative TCA methods on the IEEE 24-Bus RTS System as a bigger system to test the validity and efficiency of the proposed method.

The system is shown in Figure 4[10]. The costs of the lines are considered to be proportional to their respective series reactance. MATPOWER package version 2.0 and MATLAB 6.5 Software [8] are used to run the DC load flow. Data for generators are given in Table (16). Table (17) shows the system bus data, and Table (18) presents the line data and the line cost in U.S. *$/h*.

Table 16: IEEE 24-Bus RTS System Generation Data

Bus	P MW	Q MVAR	[illegible]	[illegible]	[illegible]	[illegible]
1	172	0	80	-50	192	62.4
2	172	0	80	-50	192	62.4
7	240	0	180	0	300	75
13	136	0	240	0	285.3	45.33
14	0	35.3	200	-50	0	0
15	215	0	210	-50	215	66.3
16	155	0	80	-50	155	54.3
18	400	0	200	-50	400	100
21	400	0	200	-50	400	100
22	300	0	96	-60	300	60
23	660	0	310	-125	660	248.6

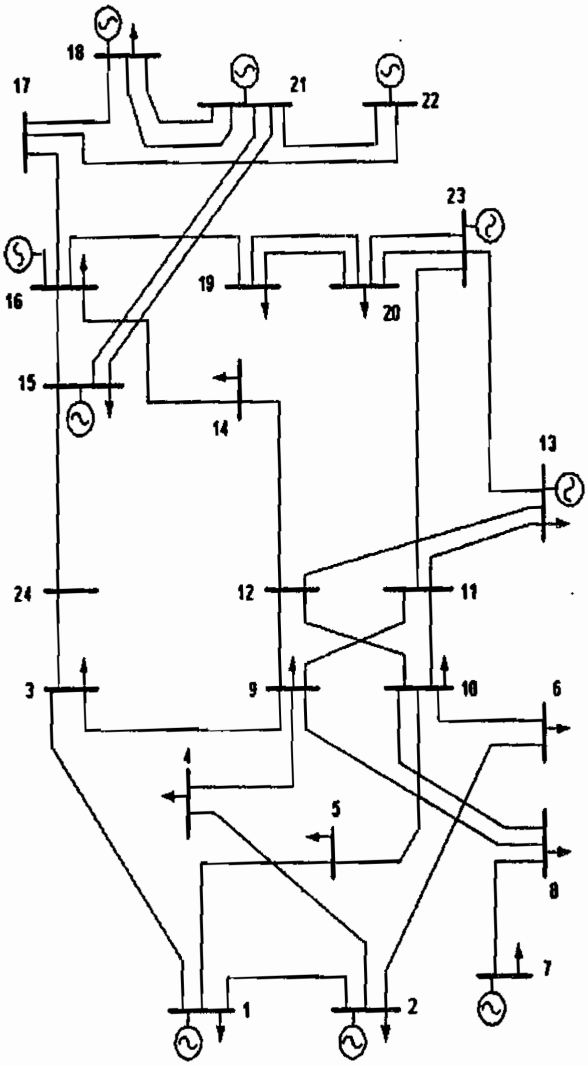

Figure 4: IEEE 24-Bus RTS System[10]

The proposed transmission cost allocation method based on superposition is applied on the system. Tables (19) and (20) show the total transmission cost allocation baseu on the superposition theorem for all the generators and demands, respectively.

Table 17: IEEE 24-Bus RTS System Bus Data

[illegible]	[illegible]	[illegible]	[illegible]
1	2	108	22
2	2	97	20
3	1	180	37
4	1	74	15
5	1	71	14
6	1	136	28
7	2	125	25
8	1	171	35
9	1	175	36
10	1	195	40
11	1	0	0
12	1	0	0
13	3	265	54
14	2	194	39
15	2	317	64
16	2	100	20
17	1	0	0
18	2	333	68
19	1	181	37
20	1	128	26
21	2	0	0
22	2	0	0
23	2	0	0
24	1	0	0

Table 18: IEEE 24-Bus RTS System Line Data

1	1	2	0.0026	0.0139	6.95
2	1	3	0.0546	0.2112	105.59
3	1	5	0.0218	0.0845	42.24
4	2	4	0.0328	0.1267	63.34
5	2	6	0.0497	0.192	95.99
6	3	9	0.0308	0.119	59.49
7	3	24	0.0023	0.0839	41.94
8	4	9	0.0268	0.1037	51.84
9	5	10	0.0228	0.0883	44.14
10	6	10	0.0139	0.0605	30.25
11	7	8	0.0159	0.0614	30.70
12	8	9	0.0427	0.1651	82.54
13	8	10	0.0427	0.1651	82.54
14	9	11	0.0023	0.0839	41.94
15	9	12	0.0023	0.0839	41.94
16	10	11	0.0023	0.0839	41.94
17	10	12	0.0023	0.0839	41.94
18	11	13	0.0061	0.0476	23.80
19	11	14	0.0054	0.0418	20.90
20	12	13	0.0061	0.0476	23.80
21	12	23	0.0124	0.0966	48.29
22	13	23	0.0111	0.0865	43.24
23	14	16	0.005	0.0389	19.45
24	15	16	0.0022	0.0173	8.65
25	15	21	0.0063	0.049	24.50
26	15	21	0.0063	0.049	24.50
27	15	24	0.0067	0.0519	25.95
28	16	17	0.0033	0.0259	12.95
29	16	19	0.003	0.0231	11.55
30	17	18	0.0018	0.0144	7.20
31	17	22	0.0135	0.1053	52.64
32	18	21	0.0033	0.0259	12.95
33	18	21	0.0033	0.0259	12.95
34	19	20	0.0051	0.0396	19.80
35	19	20	0.0051	0.0396	19.80
36	20	23	0.0028	0.0216	10.80
37	20	23	0.0028	0.0216	10.80
38	21	22	0.0087	0.0678	33.90

Table 19: Total Transmission Cost Allocated to Generators for Different TCA Methods

1	158.8729	120.42	64.27	81.44
2	163.0355	121.94	138.12	81.44
7	175.8167	161.97	30.7	113.63.
13	54.08328	76.02	0.0	117.12
15	71.28022	86.08	0.0	101.8
16	45.58571	54.21	20.36	73.39
18	145.3246	171.96	29.9	189.39
21	157.2469	166.79	309.7	189.39
22	171.4852	179.07	266.98	142.04
23	230.9891	235.24	513.69	284.08
Total	1373.72	1373.72	1373.72	1373.72

Table 20: Total Transmission Cost Allocated to Demands for Different TCA Methods

Bus				
1	89.6172	82.15	0.0	52.06
2	81.81633	75.1	0.0	46.75
3	99.47852	102.99	236.99	86.76
4	61.18796	56.95	120.66	35.67
5	51.88394	49.26	62.43	34.22
6	99.91401	94.82	192.68	65.55
7	99.2057	96.22	0.0	60.25
8	123.0226	110.35	99.82	82.42
9	83.58901	90.3	215.62	84.35
10	103.6939	104.89	218.47	93.99
13	108.1787	102.16	7.7	127.73
14	55.94025	64.37	72.77	93.51
15	89.30041	101.97	31.28	152.79
16	26.21627	29.32	0.0	48.20
18	107.1016	115.81	0.0	160.51
19	52.27256	55.26	62.24	87.24
20	41.30101	41.77	53.05	61.70
Total	1373.72	1373.72	1373.72	1373.72

4.4. Result Analysis

Table (19) shows that the SP method allocates most of the transmission cost to generators at buses 21,22and 23. This is expected because all these generators are electrically far away from load centers, and their productions are comparatively high. Comparing methods SP and EBE, it can be concluded that the SP method allocates transmission cost to generator at bus 13 more than transmission cost allocated to generator at bus 16 .On the other hand, the EBE method allocates cost to generator at bus 16 more than transmission cost allocated to generator at bus 13. Although generation at bus 16 is comparatively high because generator at bus 13 is directly connected to lines 18, 20 and 22, those have cost more than lines 23, 24, 28 and 29 which are directly connected to generator at bus 16. So, generator at bus 13 use network more than generator at bus 16. For the demands, using the SP method, the network costs are mostly allocated to demand at bus 8, as shown in Table (20). On the other hand, the EBE method allocates cost to demand at bus 18. This happens because bus 8 is located far away from the main generators: 21, 22, and 23 but, demand at bus 18 is close to heavy generations at buses 21, 22 and 23. Therefore, demand at bus 8 use many of the lines in the network.

Finally, it is observed that the SP obtained results near of the EBE results, without generation shift DF or assume stronger assumptions (the equivalent bilateral exchange principle). Also, the Pro rata (PR) and PS procedures are not advisable because they are unfair for specific groups of generators and demands. Generators close to load centers are unfairly treated with respect to generators far away from load centers. Analogously, demands close to generating areas are unfairly treated with respect to demands far away from those areas. The PR and PS methods don't take into account the network and produces substantially different results than other methods.

5. CONCLUSIONS

This paper has developed a new transmission cot allocation method based on superposition theorem. The superposition theorem allows one to consider complicated circuit in terms of simpler components. The theoretical analysis and numerical results show that the proposed method has many advantages. It is flow based method, independent on the choice of a slack bus. It is a straightforward to apply, an appropriate method that could allocate the costs of transmission services based on the actual usage by different users. It is accurate and efficient for transmission usage evaluation and fair and equitable pricing rules. Also, it is technically easy for implementation and transparent to transmission users. However, other techniques require stronger assumptions, which diminish their practical interest.

The proposed TCA method is applied on both 4-bus system and IEEE 24-bus RTS system. The results show the closeness of the proposed method result to the EBE Method which is the best among the existing TCA method. Even, our proposed one is more accurate as it is not built on assumptions and it doesn't have much burden in calculations.

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