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Two-Level PI Controller Design for Interconnected Power System Using Eigenvalues Assignment Technique.

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Two-Level and PI Controller Design for Interconnected Power System Using Eigenvalues Assignment Technique

تصميم متحكم ذو مستويين و متحكم من النوع التناسبى التكاملى لمنظومة قوى كهربية متداخلة بالإستعانة بمفاهيم القيم الذاتية

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الملخص:-
يقدم البحث طريقتين مختلفتين لتصميم متحكم لمنظومة قوى كهربية، تعتمد الطريقة الأولى على أسلوب التحكم ذو المستويين، حيث ينظر إلى منظومة القوى على أنها من ذات الأبعاد الكبيرة و مؤلفة من عدة أنظمة متداخلة و ذات أبعاد أصغر، لذا يتم تصميم المتحكمات فى المستوى الأول بأسلوب تعظيم دالة أداء و هدف لكل نظام منفصل علي حده. ثم يستخدم مفهوم و أسلوب إختيار القيم و المتجهات الذاتية لتصميم المتحكمات فى المستوى الأعلى. إذ تعبر القيم الذاتية عن كيفية الأداء و الخصائص الزمنية للمنظومة بينما تعبر المتجهات الذاتية عن شكل إستجابة المنظومة. يتميز التصميم بأسلوب التحكم ذو المستويين عموما بملائمته لنظم القوى الكهربية، حيث يتم الحفاظ لكل نظام منفصل على خصائصه. تميز أسلوب التصميم المقترح بالقيم الذاتية بإمكانية إختيار أجزاء من مكونات المتجهات الذاتية لتحقيق عدة مزايا منها المتانة. أما طريقة التصميم الثانية فى البحث الحالى توضح كيفية تصميم متحكم من النوع التناسبى التكاملى و هو شائع الإستخدام فى تطبيقات نظم القوى الكهربية المتداخلة و خاصة عند تطبيق التحكم فى التردد و ضبط الأحمال و تم ايضا الإستعانة فى التصميم بمفاهيم إختيار القيم الذاتية و ما لها من مزايا و مرونة و يقدم البحث مثالين توضيحيين لبيان جدوى و ملائمة الطريقة المقترحة.

Abstract: - The paper presents two different schemes to design controllers for interconnected power systems. The first scheme adopts two-level control concept, where the "lower" first level design is based on optimizing the decoupled subsystems, then eigenstructure (eigenvalues and partial eigenvalues) assignment approach is adopted to design the "higher" second level controller. The proposed two-level control preserves the autonomy of subsystems for sharing the assignment process, and suits power system stabilizers design. The proposed scheme can achieve robustness via eigenvector selection. The proposed second scheme proposes a proportional plus integral controller which is commonly used for load frequency control. The design procedure is based also on eigenvalues assignment. Two illustrative numerical examples are presented to prove the effectiveness of the procedure where the results indicate that the proposed control scheme works well.

Key-Words: - Load frequency control (LFC), Multi level control, Eigenvectors.

1 Introduction and Research Motivation

Electric power systems control has received a great deal of attention and will receive increasing attention in the future as a major means in the competitive energy market, and to improve performance due to the increasing size, changing structure and complexity of modern interconnected power systems. Electric power system analysis is the basis of operation and planning since the overall electric power systems are large in size and their problems are computationally complex with increasing interactions. In the dynamical operation of electric power systems, it is possible to study several issues through the multilevel or decentralized approach to facilitate several tasks and achieve requirements for stability and load frequency scheduling [1]. It is necessary that the frequency of each interconnected area and inter area tie line power are kept as near to the scheduled values as possible through effective control action and which can be achieved by designing automatic generation control (AGC). The deviation from these scheduled values are usually combined and represented in the area control error (ACE) which is computed within each area, hence providing the basis for load frequency control (LFC) action. Allowing the frequency to deviate too much from its nominal value results in a low quality of the

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delivered electric energy associated with damping vibrations in turbines. Inter-area oscillations have a direct relationship with lower frequencies. A high quality of electric power system requires both the frequency and voltage to remain at standard values during operation. An effective countermeasure to low frequency oscillations is to implement power system stabilizers (PSS), which are automatic controllers mainly of the lead-lag structure, acting on the excitation system providing additional damping leading to attenuation of oscillations under different fluctuating load conditions. A proposed algorithm for such type of stabilizers presented in [2-3] based on participation factor and decentralized pole placement to determine the PSS parameters. Another method for decentralized controller design was developed in [4], where the proposed algorithm was applied to the decentralized design of power system stabilizers for a 10-machine power system model. As an extension of the power system stabilizers, the supplementary excitation damping control (SEDC) is usually applied for damping torsional frequency oscillations [5]. Moreover, flexible AC transmission systems (FACTS) devices are used for fast voltage and reactive power control. Other possibilities for increasing damping have different controllable equipments installed in the system such as high voltage direct current (HVDC) or static VAR compensators (SVC). Several strategies for load frequency control have been introduced [6-7]. Most LFC systems use proportional plus integral (PI) based controllers, where several tuning techniques for such controllers are available based on trial and error concept [8]. Systems implementing PI controllers are capable of obtaining good dynamic performance for wide range of operating conditions and various load changes, besides the capability of PI controller to damp the undesirable oscillations and its robustness against parameter uncertainty. Load frequency control received a considerable interest in electric power system design and operation [9]. In [9] with the references therein, an overview of control strategies are provided with various methodologies with comparisons between different approaches. Recently, the LFC problem has received attention including optimal, adaptive, decentralized and intelligent control techniques [10]. However, electric power systems contain different kinds of uncertainties due to changes in load variations and errors in modeling, also the operating points of the system change randomly [11]. Dealing with parameter uncertainties through active disturbance rejection control (ADRC) for load frequency control was developed on a general transfer function [12]. Commonly at steady state operation one may linearize the set of differential equations, the algebraic equations, and the network equations to describe the system dynamic response for small deviations from the operating point. If the linearized system eigenvalues has negative real part, then the electric power system can withstand small disturbances and is considered stable in the small-signal sense. A celebrated review of power system stability problems can be found in [7].

Eigenspectrum (eigenvalues and related eigenvectors) analysis is a useful design tool in both voltage collapse analysis at each intermediate equilibrium stage of collapse (span shoots), and finding a power system operating point that is both economically optimal and stable in the small-signal sense [13]. Stability requirements may be met by adopting principle of eigenstructure assignment or optimal control techniques, taking into consideration that the need for disturbance rejection creates the need for feedback control.

For the purpose of electric power system control design, the present paper considers designing feedback controller based on eigenstructure assignment as a tool due to its numerous beneficial advantages since eigenvalues govern the rate of decay of various portions of the system dynamic response, while eigenvectors share prescribing the shape of closed loop response.

Multilevel control has been an accepted approach in many control applications. A computationally efficient constructive procedure for a two-level controller design is presented for controlling fully interconnected electric power system to ensure an improved prescribed performance. Also the decentralized LFC and frequency control in emergency conditions based on singular value theory for multi area power system was addressed in [14]. This paper proposes a two-level control structure based on eigenstructure properties to ensure an improved prescribed performance using the algebraic relations between the system model and a targeted closed loop eigenstructure.

This paper is organized as follows; the introduction presents the research significance and motivation. Section two proposes a multilevel controller where the lower level controller is based on optimizing an associated index with its own criterion, while the higher level is based on achieving a prescribed spectrum. Section three presents a proportional plus integral controller that suits a power system load frequency control. Two illustrative numerical examples demonstrate the significant advantage of the proposed procedure.

2 Proposed Multilevel Controller Design

From the control theory point of view, the electric power system is considered as a large-scale interconnected system. In this section, a two-level controller structure commonly used for large-scale interconnection is presented.

In an interconnected electric power system, the centralized analysis and control approach are infeasible due to the requirement of extensive amount of shared information exchange, complexity, and high dimensionality. To cope with such systems, several methodologies have been applied where most of them embrace the decentralization and decomposition approaches [15-16]. The state space linearized representation of the interconnected system under consideration composed of N coupled subsystems (multimachines) can be represented as:

$$\dot{x}_r(t) = A_r\, x_r(t) + \sum_{\substack{q=1\\ \neq r}}^{N} A_{rq}\, x_q(t) + B_r\, u_r(t) \quad (1)$$

where $x_r(t) \in \Re^{n_r}$ and $u_r \in \Re^{m_r}$ are the state and control vectors associated with the r-th subsystem (r=1,...,N). $A_r \in \Re^{n_r \times n_r}$ is a constant system matrix, and matrix $A_{rq} \in \Re^{n_r \times n_q}$ reflects the strength of interaction for the integral signals and exchange of information among subsystems r and q tied by high voltage transmission or tie lines, while $B_r \in R^{n_r \times m_r}$ is the constant control matrix.

The objective is to design a two-level controller to achieve a prespecified self conjugate spectrum for the interconnected subsystem such that all subsystems contribute to satisfy the desired spectrum. An approach to stabilize power system transients based on shifting the dominant eigenvalues via optimization procedure was presented in [13]. The present proposed controller configuration is of the multilevel structure commonly used for large-scale interconnected systems. The underlying multilevel computation structure has several features [1]. Among these are the increased reliability and flexibility that is needed for the concerned electric power system. Let the controller gain be composed of two levels as:

$$u^l(t) = diag\,(K_r)\, x_r(t) \quad (2)$$

where $K_r \in \Re^{m_r \times n_r}$ is the lower "local" subsystem level controller and

$$u^g(t) = \sum_{\substack{q=1\\ \neq r}}^{N} K_{rq}\, x_q(t) \quad (3)$$

where $K_{rq} \in \Re^{m_r \times n_q}$ is the global "higher" level controller representing feedback from other areas. Therefore the two-level control associated with subsystem (r) is $u_r(t)$ consisting of two components namely:

$$u_r = u_r^l + u_r^g \quad (4)$$

Both components are working together achieving the global objective. The resulting closed loop is given as:

$$\dot{x}_r(t) = (A_r + B_r\, K_r)\, x_r(t) + \sum_{\substack{q=1\\ \neq r}}^{N} (A_{rq} + B_r\, K_{rq})\, x_q(t) \quad (5)$$

A decentralized "local" controller based on optimizing decoupled linear quadratic performance index associated with each subsystem is proposed for the lower level. Many methods for control design are based on optimization techniques considering a diverse performance index that each has its merits and limitations [16-17]. The lower level feedback gain is obtained according to optimizing a linear quadratic performance index with respect to $u_r(t)$ as:

$$J_r(t_0, x_{r0}(t), u_r(t)) = \int_0^{\infty} (x_r^T(t)\, Q_r\, x_r(t) + u_r^T(t)\, R_r\, u_r(t))\, dt \quad (6)$$

For the free r-th subsystem

$$\dot{x}_r(t) = A_r\, x_r(t) + B_r\, u_r(t) \quad (7)$$

where $0 \le Q_r \in \Re^{n_r \times n_r}$ and $0 < R_r \in \Re^{m_r \times m_r}$ are symmetric positive semi definite and positive definite weighting matrices respectively. Under the assumption of complete controllability of each subsystem (A_r, B_r), there exists a unique linear optimal control $u_r^*(t)$ which can be expressed as:

$$u_r^*(t) = -R_r^{-1}\, B_r^T\, P_r \quad (8)$$

where $0 < P_r \in \Re^{n_r \times n_r}$ is the symmetric unique solution of the algebraic Riccati equation given as:

$$A_r^T\, P_r + P_r\, A_r + Q_r - P_r\, B_r\, R_r^{-1}\, B_r^T\, P_r = 0 \quad (9)$$

Such $u_r^*(t)$ yields the minimum of performance index (6), given as:

$$J_r^*(t_0, x_{r0}) = x_{r0}^T P_r x_{r0} \quad (10)$$

It should be mentioned that the above design simulates the design of PSS. Furthermore, the optimal control signal $u_r^*(t)$ provides stability to the matrices $(A_r - B_r R_r^{-1} B_r^T P_r)$ $(r=1,\ldots,N)$.

Hence, the overall local gain (K^L) is given as:

$$K^L = blockdiag\ (-R_r^{-1} B_r^T P_r) = blockdiag(k_r) \quad (11)$$

To illustrate the proposed procedure, consider the interconnected system extracted from [15] where:

$$A = \begin{bmatrix} A_1 & A_{12} \\ A_{21} & A_2 \end{bmatrix}, A_1 = \begin{bmatrix} 6 & -10 \\ 10 & 20 \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} -0.3 & 0.4 & 0.2 \\ -0.1 & 0.1 & 0.1 \end{bmatrix}, A_{21} = \begin{bmatrix} -20 & -10 \\ 5 & 4 \\ 1 & -1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 10 & 5 & 8 \\ 2 & -10 & 2 \\ 10 & 0 & -2.5 \end{bmatrix}, \quad B = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.1 & 0.25 \\ -1 & 1 \end{bmatrix}, B_2 = \begin{bmatrix} 0.1 & 0.25 \\ -1 & 1 \\ 1 & 1 \end{bmatrix}$$

Interestingly, each decoupled (free subsystem) is controllable but unstable with eigenvalues as $(13 \pm j\,7.1414)$ for the first subsystem (A_1, B_1) and $(15.1542, -8.2092, -9.4450)$ for subsystem (A_2, B_2). Increasing the values of the weighting matrices Q_r will increase the overshoot of the corresponding state variable, while decreasing the settling time. Meanwhile, increasing the values of the weighting matrices R_r reduces the control effort. For the quadratic performance index given in (6) applied on two subsystem aforementioned in (7) with $Q_1 = I_2$, $R_1 = I_2$, $Q_2 = I_3$ and $R_2 = I_2$. Solving the algebraic Riccati equation shown in (9) using Matlab® results in;

$$P_1 = \begin{bmatrix} 43.3384 & -12.2327 \\ -12.2327 & 26.2991 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 30.7260 & 6.0824 & 14.4828 \\ 6.0824 & 1.2540 & 2.8659 \\ 14.4828 & 2.8659 & 6.9101 \end{bmatrix}$$

Whereas the local optimal gains are:

$$k_1^l = \begin{bmatrix} 16.5666 & -27.5223 \\ -1.3981 & 23.2409 \end{bmatrix}$$

while $k_2^l = \begin{bmatrix} 11.4730 & 2.2201 & 5.4925 \\ 28.2467 & 5.6405 & 13.3967 \end{bmatrix}$.

The overall local gain $K^L = \begin{bmatrix} k_1^l & 0 \\ 0 & k_2^l \end{bmatrix}$.

Consequently, the closed-loop optimal control subsystems are:

$(A_1 - B_1 k_1^l) = \begin{bmatrix} 4.6929 & -13.0580 \\ 27.9647 & -30.7632 \end{bmatrix}$ where its optimal eigenvalues became $(-13.0352 \pm j\,7.133)$ while

$$(A_2 - B_2 k_2^l) = \begin{bmatrix} 1.7910 & 3.3679 & 4.1016 \\ -14.7738 & -13.4204 & -5.9042 \\ -29.7197 & -7.8606 & -21.3891 \end{bmatrix}$$

where its optimal eigenvalues became $(-15.1857, -8.3193, -9.5135)$.

The global "higher level" gain $K^G = [k_{rq}]$ $(r,q=1,\ldots,N,\ r\neq q)$ is obtained via eigenstructure assignment procedure.

The decoupled "lower level" optimal subsystems are represented as:

$$\dot{x}_r(t) = (A_r - B_r K_r)\, x_r(t) \quad (12)$$

The decoupled overall system is given as:

$$\dot{x}(t) = A_o\, x(t) \quad (13)$$

where

$$x(t) = [x_1^T(t), x_2^T(t), \ldots, x_N^T(t)]^T \in \Re^n, (n = \sum_{r=1}^{N} n_r)$$

is the overall state vector which uniquely describe the system while

$$A_o = blockdiag\ (A_r - B_r K_r) \in \Re^{n_r \times n_r} \quad (14)$$

The overall control vector is given as:

$$u(t) = [u_1^T(t), u_2^T(t), \ldots, u_N^T(t)]^T \in \Re^m, (m = \sum_{r=1}^{N} m_r) \quad (15)$$

Using equations (2) and (3), the composite control vector can be expressed as:

$$u(t) = \left(K^L + K^G\right) x(t) = K\, x(t) \quad (16)$$

Therefore the composite closed loop will be on the form of:

$$\dot{x}(t) = (A_o + \overline{A} + B\,K)\, x(t) \quad (17)$$

where $\overline{A} = [A_{rq}] \in \Re^{n \times n}$ (18)

$B = blockdiag\ (B_r) \in \Re^{m \times n}\ (r,q=1,\ldots,N,\ r \neq q)$ (19)

Let $\Lambda = \{\lambda_i, i = 1,2,\ldots, n\}$ represents a desired closed loop self conjugate set of eigenvalues representing a desired (improved) dynamic response specification. There exists at least one nonzero vector $V_i \in \Re^n$ such that

$$(A_o + \bar{A} + B\,K - \lambda_i\, I)\, V_i = 0 \qquad (20)$$

The proposed procedure is based on mode separation by decomposing the overall eigenvector (V_i) into subeigenvectors associated with each decoupled subsystem. It is possible to simplify the composite relation (20) to the subsystems level as:

$$v_i = [v_{i1}^T, v_{i2}^T, \ldots, v_{iN}^T]^T,\ v_{ir} \in \Re^{n_r} \qquad (21)$$

This decomposition yields:

$$\left[(A_r - B_r\, K_r^l - \lambda_i\, I)\, v_{ir} + \sum_{\substack{q=1 \\ \neq r}}^{N} (A_{rq} + B_r\, K_{rq})\, v_{iq} \right] = 0 \qquad (22)$$

It may be represented as:

$$[(A_r - B_r\, K_r^l - \lambda_i I) \quad A_{rq} \quad \ldots\ldots \quad A_{rN} \quad B_r]_{n_r \times (n+m_r)} \times \begin{bmatrix} v_{ir} \\ v_{iq} \\ \ldots\ldots \\ v_{iN} \\ \sum_{\substack{q=1 \\ \neq r}}^{N} k_{rq}\, v_{iq} \end{bmatrix}_{(n+m_r)\times 1} = 0 \qquad (23)$$

Equation (23) represents (n_r) linear equation in (n + m_r) parameter forming the rightmost vector.
One can freely select (n - (n_r - m_r)) element including the lower most (m_r) vector w_{ri}.

$$w_{ri} = \sum_{\substack{q=1 \\ \neq r}}^{N} k_{rq}\, v_{iq} \qquad (24)$$

Then repeating the selection for (i =1,..., n), and arranging to get

$$W_r = [w_{r1} \quad w_{r2} \quad \ldots\ldots \quad w_{rn}] \in \Re^{m_r \times n} \qquad (25)$$

Also, repeating for all subsystems and arranging it as:

$$\begin{bmatrix} w_{11} & w_{12} & \ldots\ldots & w_{1n} \\ w_{21} & w_{22} & \ldots\ldots & w_{2n} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ w_{N1} & w_{N2} & \ldots\ldots & w_{Nn} \end{bmatrix} = \begin{bmatrix} 0 & k_{12} & \ldots\ldots & k_{1N} \\ k_{21} & 0 & \ldots\ldots & k_{2N} \\ \ldots\ldots & \ldots\ldots & 0 & \ldots\ldots \\ k_{N1} & k_{N2} & \ldots\ldots & 0 \end{bmatrix} \times \begin{bmatrix} v_{11} & v_{21} & \ldots\ldots & v_{n1} \\ v_{12} & v_{22} & \ldots\ldots & v_{n2} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ v_{1N} & v_{2N} & \ldots\ldots & v_{nN} \end{bmatrix} \qquad (26)$$

Let the desired closed-loop self conjugate set of eigenvalues to be assigned to the composite system via local and global control be $(-0.1 \pm j\,0.1, -0.2 \pm j\,0.3, -1)$.

In order to obtain the higher level gains k_{12} and k_{21} while taking into account interaction matrices A_{12} and A_{21}, equation (23) is applied for $r = 1, \lambda_1 = -0.1 + j\,0.1$ which yields:

$$v_{11} = -\left(A - B_1 k_1^l - \lambda_1 I\right)^{-1} B_1 w_{11} - \left(A - B_1 k_1^l - \lambda_1 I\right)^{-1} A_{12} v_{12}$$

Numerical values are:

$$\begin{bmatrix} 4.7929 - j0.1 & -13.0580 & 0.3 & 0.4 & 0.2 & 0.1 & 0.25 \\ 27.9647 & -30.6632 - j0.1 & -0.1 & 0.1 & 0.1 & -1 & 1 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{12} \\ w_{11} \end{bmatrix} = 0$$

Arbitrarily selecting w_{11} as $\begin{bmatrix} 0.3 \\ 0.6 \end{bmatrix}$ and v_{12} as $\begin{bmatrix} 1+j \\ 0.2-j \\ 0.7-j\,0.3 \end{bmatrix}$ while forming the lower five elements of the right most vector, one gets v_{11} as $\begin{bmatrix} -0.0226 + j\,0.0787 \\ -0.003 - j\,0.0277 \end{bmatrix}$.

Meanwhile for $\lambda_2 = -0.1 - j\,0.1$, selecting w_{12} = conjugate (w_{11}) and v_{22} = conjugate (v_{12}); then v_{21} results as conjugate to v_{11}. Repeating the procedure for r = 1,2, and i = 1,...,5 to form equation (26), the desired high level gains are expressed as:

$$\underset{m_1 \times n_2}{k_{12}} = \begin{bmatrix} -2.1764 & 0.6485 & 13.1600 \\ 5.6950 & 2.8137 & 6.1087 \end{bmatrix}$$ and

$$\underset{m_2 \times n_1}{k_{21}} = \begin{bmatrix} -20.6243 & 27.5748 \\ -17.2874 & 10.8792 \end{bmatrix}$$ where the higher level gain $K^G = \begin{bmatrix} 0_{2\times 2} & k_{12} \\ k_{21} & 0_{2\times 3} \end{bmatrix}$ while the overall gain that assigns the desired spectrum is $\left(K^L + K^G\right)$.

An interesting advantage of the proposed procedure is that the eigenvectors can be partially selected. The detailed selection procedure had been previously presented for a centralized system [18]. Relation (26) is expressed in a compact form as:

$$W = K^{G} V \quad (27)$$

where matrix $W \in \Re^{m\times n}$ contains all the properly selected free parameters while $V \in \Re^{n\times n}$ is the eigenvector matrix.

The global gain K^{G} is obtained from (27) as:

$$K^{G} = W V^{-1} \quad (28)$$

It should be pointed out that the design complexity is resolved via partioning the overall eigenvector as shown in (21). Robustness property for the interconnected systems is a desired approach in decentralized control. An extra design freedom can be exploited by shaping the closed loop transient response by selecting the matrix W in equation (26) that results in as orthogonal as possible eigenvector such that $V^{-1} = V^{T}$ (29)

The more robust the closed loop system is, the better the design will be. This condition assures robust performance. Combining the local level and global level controllers, equations (11) and (28), the controller structure is expressed in (16). An amazing property for the proposed procedure is that as the interconnections are disconnected from each other, subsystems behave optimally.

3 PI Controller Design

To set the frequency and tie line power back to their set values, PI controller can be used. This section presents a comprehensive PI controller design procedure based on eigenvalues assignment as a tool by exploiting the non unique feedback gain for the multivariable augmented system. Let the linearized controllable observable state space representation of a two-control area power system be represented as:

$$\dot{x}(t) = A\,x(t) + B\,u(t) + L\,w(t) \quad (30.a)$$

$$y(t) = C\,x(t) \quad (30.b)$$

where $x(t) \in \Re^{n}$ is the state vector composed of measurable components. The matrices A, B, C and L are constants and of appropriate dimensions. $u(t) \in \Re^{m}$ is the two-input control vector, $w(t)$ is the m-dimensional unknown but constant step load or slowly varying disturbance signals, $y(t) \in \Re^{q}$ is the overall output vector. It is logical to assume that step change of load can be considered as a disturbance; hence in order to achieve the finite constant load disturbance rejection $w(t)$, an effective disturbance rejection controller is designed by differentiating equation (30.a) as:

$$\dot{z}(t) = A\,z(t) + B\,\dot{u}(t) \quad (31)$$

where $z(t) = \frac{dx}{dt}$. Defining an augmented state vector $x_a(t) = \begin{bmatrix} z^{T}(t) & y^{T}(t) \end{bmatrix}^{T}$ where $x_a(t) \in \Re^{n+q}$. Therefore, the augmented system can be written as:

$$\dot{x}_a(t) = A_a\,x_a(t) + B_a\,\dot{u}(t) \quad (32.a)$$

$$y_a(t) = C_a\,x_a(t) \quad (32.b)$$

where

$$A_a = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}_{(n+q)\times(n+q)}, B_a = \begin{bmatrix} B \\ 0 \end{bmatrix}_{(n+q)\times m}, C_a = \begin{bmatrix} C & 0 \end{bmatrix}_{q\times(q+n)} \quad (33)$$

The objective is to design an augmented PI control which takes the form:

$$u(t) = K_P\,x(t) + K_I \int_0^{\infty} y(t)\,dt \quad (34.a)$$

Differentiating both sides to obtain $\dot{u}(t)$:

$$\dot{u}(t) = K_P\,z + K_I\,C\,x = K_a\,x_a(t) \quad (34.b)$$

such that the non unique gain $K_a = \begin{bmatrix} K_P & K_I \end{bmatrix} \in \Re^{m\times q}$ where $K_P \in \Re^{m\times n}$ represents the proportional part, while $K_I \in \Re^{m\times q}$ represents the integral part. Appling equation (34), results in:

$$\dot{x}_a(t) = (A_a + B_a\,K_a)\,x_a(t) \quad (35)$$

The design procedure is summarized as follows which is based on a procedure represented in [19].

To achieve a prespecified symmetric self conjugate spectrum $\Lambda = \{\lambda_i, i = 1,2,\ldots, n+q\}$ for the augmented system to provide a desirable transient response through PI controller, the following relations are satisfied:

$$\left|A_a + B_a\,K_a - \lambda_i\,I\right| = 0 \quad (36)$$

Then,

$$\left|(A_a - \lambda_i\,I)\right|\left|I_{n+q} + (A_a - \lambda_i\,I)^{-1}\,B_a\,K_a\right| = 0 \quad (37)$$

Let $\Phi_a(\lambda_i) \equiv (A_a - \lambda_i\,I)^{-1}$, then equation (37) reduces to:

$$\left|(A_a - \lambda_i\,I)\right|\left|I_{n+q} + \Phi_a(\lambda_i)\,B_a\,K_a\right| = 0 \quad (38)$$

Applying the well known identity $\left|I_{n+q} + P\,Q\right| = \left|I_m + Q\,P\right|$ where P is $(n+q)\times m$

matrix while Q is $m\times(n+q)$ matrix. Consequently, equation (38) is transformed to:

$$\left|A_a - \lambda_i\, I_{n+q}\right|\left|I_m + K_a\,\Phi_a(\lambda_i)\,B_a\right| = 0 \quad (39)$$

This relation indicates that there must not be a common eigenvalue between open and closed loop systems.

One possible solution for (39) is the sufficient condition

$$K_a\,\Psi_a(\lambda_i) = -I_m \quad (40)$$

where $\Psi_a(\lambda_i) = \Phi_a(\lambda_i)\,B_a$ is an $(n+q)\times m$ matrix of rank (m) associated with the eigenvalue λ_i. Therefore in total it contains $(n+q)\times m$ linearly independent columns. The j-th column of the left hand side of equation (40) and the j-th column of I_m satisfies the relation:

$$K_a\,\Psi_a(\lambda_i) = -e_j \quad (41)$$

As the desired eigenvalues are distinct, the columns of the $(n+q)\times(n+q)$ matrix $[\Psi_a(\lambda_1)\;\;\Psi_a(\lambda_2)\;\;.....\;\;\Psi_a(\lambda_{n+q})]$ are linearly independent. (42)

Selecting $(n+q)$ columns from equation (42) representing the contribution of all the prescribed eigenvalues in the design process to form the $(n+q)\times(n+q)$ matrix denoted as:

$$T = [\Psi_{aj_1}(\lambda_1)\;\;\Psi_{aj_2}(\lambda_2)\;\;.....\;\;\Psi_{aj_{(n+q)}}(\lambda_{n+q})] \quad (43)$$

Consequently, the corresponding columns e_j in equation (41) are arranged in $m\times(n+q)$ matrix as:

$$E = [e_{j1}\;\;e_{j2}\;\;.....\;\;e_{j(n+q)}] \quad (44)$$

Accordingly, the desired PI controller gain (K_a) is obtained as:

$$K_a\,T = -E \quad (45)$$

Hence $K_a = -E\,T^{-1}$ (46)

The available flexibility in the choice of columns that form equation (43) can be exploited to satisfy several requirements (e.g. minimum norm gain, robustness, improved eigenvalues sensitivity and shifting poorly damped eigenvalues).

4 Illustrative Example

The significant advantage of the procedure is demonstrated via the following numerical example. Considering a two-area power system as a case study with data taken from the well known model [20] such that:

$$A = \begin{bmatrix} -0.05 & 6 & 0 & -6 & 0 & 0 & 0 \\ 0 & -3.33 & 3.33 & 0 & 0 & 0 & 0 \\ -5.21 & 0 & -12.5 & 0 & 0 & 0 & 0 \\ 0.45 & 0 & 0 & 0 & -0.545 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.05 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3.33 & 3.33 \\ 0 & 0 & 0 & 0 & -5.21 & 0 & -12.5 \end{bmatrix} \quad (47.a)$$

$$B = \begin{bmatrix} 0 & 0 & 12.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 12.5 \end{bmatrix}^T \quad (47.b)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 1 \end{bmatrix} \quad (47.c)$$

The open loop system eigenvalues (electromechanical modes) are

$$(-0.8312 \pm j\,2.8855, -1.2479 \pm j\,2.4743, -0.9386, -13.2789, -13.2843)$$

As long as the pair (A, B) is completely state controllable, it is possible to assign all the electromechanical modes to specified desirable locations to avoid poorly damped oscillations. The state vector associated with matrix (47.a) is defined as:

$[\Delta P_{tie}\;\;\Delta f_1\;\;\Delta P_{g1}\;\;\Delta x_{v1}\;\;\Delta f_2\;\;\Delta P_{g2}\;\;\Delta x_{v2}]$, the control vector associated with (47.b) are $[\Delta P_{c1}\;\;\Delta P_{c2}]$, while the output vector associated with (47.c) are $[(ACE)_1\;\;(ACE)_2]^T$ where ΔP_{tie} is the interconnection tie-line power, Δf is the incremental frequency deviation, ΔP_g is the incremental power generation level, Δx_v is the incremental change in valve position, ΔP_c is the incremental change in speed changer position and ACE is the area control error.

Let the desired closed loop time response, which the power system is required to track, be expressed as the location of the closed loop eigenvalues for the system expressed in (33) specified as:

$$(-0.69 \pm j\,0.69, -0.91 \pm j\,0.91, -2.38 \pm j\,2.78, -3.06 \pm j\,2.5, -24)$$

The significance of these eigenvalues are defined for both low and high frequency roots with different damping measures. Applying the proposed PI design procedure using MATLAB software results in the proportional and integral feedback gains as:

$$K_P = \begin{bmatrix} -1.3269 & -1.8008 & -1.1408 & 1.3982 & 1.312 & 1.5784 & 1.5345 \\ 0 & 0 & 0 & 0.1049 & 0.2724 & -0.3051 & 0.6272 \end{bmatrix}$$

$$K_I = \begin{bmatrix} -1.1182 & 1.6051 \\ -0.0472 & -0.0472 \end{bmatrix}$$

5 Conclusion

This paper considered two proposed controller design procedures for interconnected power system load frequency problem. Due to poor choice of primary controller parameters and in order to have a good closed loop performance, a prescribed set of eigenvalues have been specified to provide a desirable response. The first design procedure adopts the decentralized "two-level" control strategy. The local level controller design is based on index optimization that simulates power system stabilizer design; while the higher level controller has been designed based on eigenspectrum assignment (eigenvalues and partial eigenvectors). Viewing interconnected power system as a group of locally optimized subsystems, and globally interconnected controllers tuned through eigenstructure assignment can guarantee the overall stability and suboptimality. An interesting feature of the procedure is that one can partially select the eigenvectors to shape the output response without scarifying robustness. The objective of the second procedure is to design a proportional plus integral controller commonly used for power system load frequency control. To set the frequency and tie-line power back to their set values based on eigenvalues assignment procedure which is flexible to select the nonunique controller to achieve a desirable performance. A two-area power system is used as a case study.

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