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A. El-Zahaby
Mechanical Power Engineering Department., Faculty of Engineering., Tanta University., Egypt.

M. Bassiouny
Mechanical Power Engineering Department., Faculty of Engineering., Menofia University., Egypt.

A. Khalil
Mechanical Power Engineering Department., Faculty of Engineering., Tanta University., Egypt.

E. El-Shenawy
Mechanical Power Engineering Department., Faculty of Engineering., Tanta University., Egypt.

E. El-Agouz
Mechanical Power Engineering Department., Faculty of Engineering., Tanta University., Egypt.

See next page for additional authors

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CHARACTERISTICS OF PLAN POISEUILLE FLOWS UNDER THE EFFECT OF ELECTRO-MAGNETICS FIELD

خصائص سريان بوازيل تحت تأثير المجالات الكهرومغناطيسية

A.M. El-Zahaby[a], M. K. Bassiouny[b], A. Khalil[a], E.A. El-Shenawy[a], E. A. El-Agouz[a] and M. I.Amro[a]

a- Mechanical power engineering dept., Faculty of engineering, Tanta University, Egypt

b- Mechanical power engineering dept., Faculty of engineering, Menofia University, Egypt

ملخص البحث:

تم تقديم حلاً تحليليا للسريان كامل الطور بين سطحين متوازيين لمائع موصل للتيار الكهربى تحت تأثير المجالات الكهرومغناطيسية. و تم إثبات قيم كلا من السرعة و معدل السريان و إجهاد القص و معامل الاحتكاك الطبقى عند الأسطح.

لقد لوحظ أن القوى الكهرومغناطيسية الناتجة عن المجالات قادرة على قيادة السريان فى حال عدم وجود قيمه لتدرج الضغط . و أيضا هذه القوى تعمل بشكل كبير على تغيير خصائص و أسلوب السريان فى وجود تدرج الضغط. و لقد تم إيجاد التغير فى السرعة و إجهاد القص و معامل الاحتكاك الطبقى للسريان كدوال فى كل من النسبة الهندسية بين الارتفاع بين اللوحين و اتساع الأقطاب الكهربية، كثافة التيار الكهربى، كثافة الفيض المغناطيسى.

Abstract:

The fully developed flow between two parallel plates in an electrically conducting fluid under the acting of electromagnetic fields is solved analytically. The value of flow rate. velocity. shear stress and skin friction coefficient at the plates are derived. It is noticed that the electromagnetic forces are able to derive the flow in the absence of the pressure gradient also; the flow characteristics are largely changed when the electromagnetic fields are applied in the presence of the pressure gradient. The change in velocity. shear stress and skin friction coefficient are represented as a function of the geometrical ratio between the channel height and the electrode width h/a, electric current density and strength of magnetic field.

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NOMENCLATURES

A,B	constants of velocity Eq.38 defined by Eq.39
B_o	applied magnetic field, tesla
C_f	skin friction coefficient, --- defined by Eq.33
E	external electric field V/m
E_f	electric field parameter
g	gravity acceleration, m/s^2
H	channel height , m
J_o	electric current density , amp/m^2
L_f	Lorentz force, N
p	pressure, pa
P_{EM}	Electromagnetic parameter, ---- defined by Eq. 13
Q	discharge m^3/s
R_e	Reynolds's number,---
U	dimensionless velocity,---
u	velocity, m/s
U_{avg}	average velocity, m/s defined by Eq.18
U_p	velocity due to hydrodynamic pressure gradient m/s, defined by Eq. 9
U_z	velocity due to electromagnetic force, m/s, defined by Eq. 10
x, y, z	dimensional coordinates, m
X,Y, Z	dimensionless coordinates, ---
Greek symbols	
μ	dynamic viscosity $N.s/m^2$
ν	kinematics viscosity m^2/s
ρ	density Kg/m^3
σ	electrical conductivity 1/(Ohm.m)
τ	shear stress, N/m^2, defined by Eq. 25
τ_o	reference shear stress N/m^2, defined by Eq.26
Subscripts	
EHD	electrohydrodynamics
MHD	magnetohydrodynamic
EMHD	electromagnetohydrodynamics

1. Introduction:

Prandtl was the first one try to control the wall bounded flow. This when he used a trip wire to trigger transition in the boundary layer, generally, all competition to control the wall bounded flow, towards to reduce viscous stresses in boundary layer. Since Prandtl experiments, many numerical and experimental studies have been performed to detect and determine the efficient and most suitable control mechanisms to overcome the wall shear stress and turbulence.

It is found that most of these events linked to the boundary layer which the low speed streak and 3D vortices structure known as hairpins structures street produced by a wall-hemisphere as mentioned by Du and Karniadakis,[1] and Rossi and Thibault,[2]. If the fluid is electrically conducting, the using of magnetic and electric fields to control the flow separation and reducing drag is a favorable method.

There are three general approaches for utilizing the electric field for modifying the flow characteristics and flow control; first the electric field was created as an induced current from the coupling between the applied magnetic field and flow velocity. This current coupling with the magnetic field itself and producing an electromagnetic body force which known as Lorentz force. In this case the phenomenon is called magnetohydrodynamic MHD as mentioned by Ko and Dulikravich, [3]. By this method, we able only to control the magnitude of the producing force but the force direction can't be controlled since it is always opposing the flow direction. Liquid metals, because of their high conductivity (although it does drop with an increase in temperature), have a variety of MHD applications including the cooling of nuclear reactors, as a self cooling for the liquid sodium blanket. As studied by Kim and Abdou, [4] and Takeuchi et al, [5] .

Magnetohydrodynamics was originally developed to describe several phenomena surrounding the behavior of plasma around the earth and in space (e.g., aurora borealis, interstellar medium dynamics). Eventually, research into plasmas and electromagnetic fields grew and formed a variety of multidisciplinary engineering applications. Plasma has been used in an increasing number of industrial applications, with the most notable new product being plasma displays for panel televisions. The knowledge of magnetohydrodynamics has produced advances in utilizing plasma, alternative power generation, weaponry, and propulsion.

As studied by Braun et al, [6] although of all important applications of MHD, but if the liquids used have a weakly electrical conductivity, it lost most of its benefits.

and ionized gases is so small to achieve the required purposes. This because the weakly conducting fluid needs to external electric field to obtain a Lorentz force able to modify its flow characteristics.

Secondly, the using of electric field only without any magnetic field, the phenomenon is called electrohydrodynamics EHD in which the electric fields applied with a high voltage to create plasma flow. The plasma either accelerates or decelerates the flow depending on the electrodes arrangement. This modifying flow characteristics and enhancement the heat transfer. This could be founded in Dulikravich and Colaco, [7].

Another and new way attention is givens towards the effect of both electric and magnetic fields as external fields, this is the third approach. The phenomenon by this way is known as Electromagnetohydrodynamics EMHD, the Lorentz force creating in this case independent on the flow characteristics so, enabling us to control the flow by accelerating or decelerating its velocity depending on the direction of the creating force which define by the right hand basic for Fleming. This could be found on Rossi et al [8] and [9]. The using of Lorentz force to reduce shear stress, drag reduction and delay or totally delete the flow separation and modify the boundary layers on flat plate, and create new boundary layers was discussed by many users as Rossi and Thibault, [10] and Mutschke et al, [11].

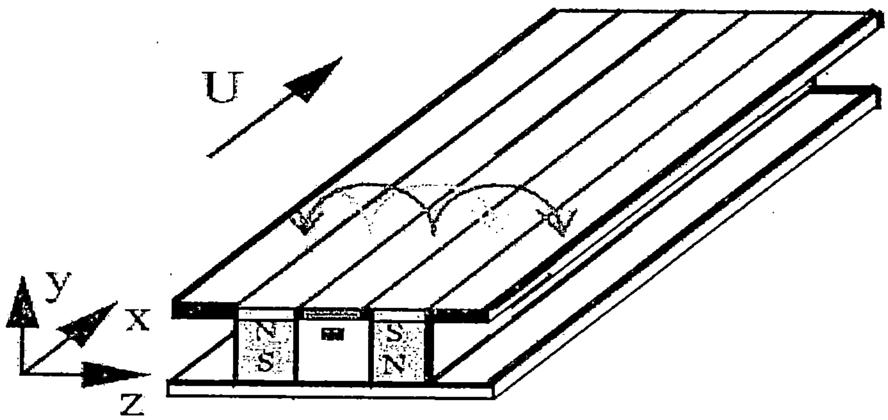

Fig., (1) details of the surface contain the electromagnetic actuator

The force using in controlling the flow is known as exponentially decaying Lorentz force, this is due to its effect is maximum at its creating surface and decreasing exponentially with the height from that surface. In order to create this force, the electric (+/-) electrodes and permanent magnets (N/S) poles must be combined on one plate at the wall surface. This combination is known as electromagnetic electric (+/-) electrodes and permanent magnets (N/S) poles must be combined on one plate at the wall surface. This combination is known as electromagnetic actuator or Riga plate, as illustrated in Fig.1, the arrow for velocity represent the electromagnetic Lorentz force and the flow direction. This found in Weier et al, [12] and Berger et al, [13].

There are two methods to apply this force to control the flow, called the wall parallel or streamwise Lorentz force and the wall normal or spanwise Lorentz force. The using of Lorentz force in controlling the flow, decrease shear stress, drag reduction and deleting flow separation comes from the principle that these forces affecting in the near wall and boundary layer regions and the most of the turbulence and eddies events noticed in this region, this could found in Breuer and Park, [14].

The attention to use the Lorentz force creating by both external magnetic and electric fields is started to be in focus of study by Nosenchuck et al,1995 [15], and Albrecht et al, [16].

Continuous studies up till now is founded toward to flow control using Lorentz force is conducting at the institute for Aerospace Engineering in Dresden, Germany as studied Weier et al, [17] and Cierpka et al, [18].

Although most investigations have been done in different fields as fusion of metals and control of flow through nuclear reactors blankets, but still many of simple flow problems have not been investigated until now. In this paper, the flow between two parallel plates is derived under the effect of Lorentz force with analyzing its boundary layers characteristics.

2. Mathematical Model:

Consider the 2D flow between two horizontal; infinite, parallel plates and the flow along the plate where u and v denoting the velocity components in the x and y directions respectively, where x is the coordinate along the plats and y perpendicular to x. Assuming that the electromagnetic fields exists at the lower plates so, the fluid is forced to move due to the force action and the flow is assumed to be two dimensional flow and fully developed, with constant fluid properties. Weier et al, [12].

2.1Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

2.2 Momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]+$$
$$\frac{\pi J_o B_o}{8\rho}\exp[\frac{-\pi}{a}(y+h/2)]+ \qquad (2\text{-a})$$
$$\frac{\pi J_o B_o}{8\rho}\exp[\frac{\pi}{a}(y-h/2)]$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\upsilon\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right] \qquad (2\text{-b})$$

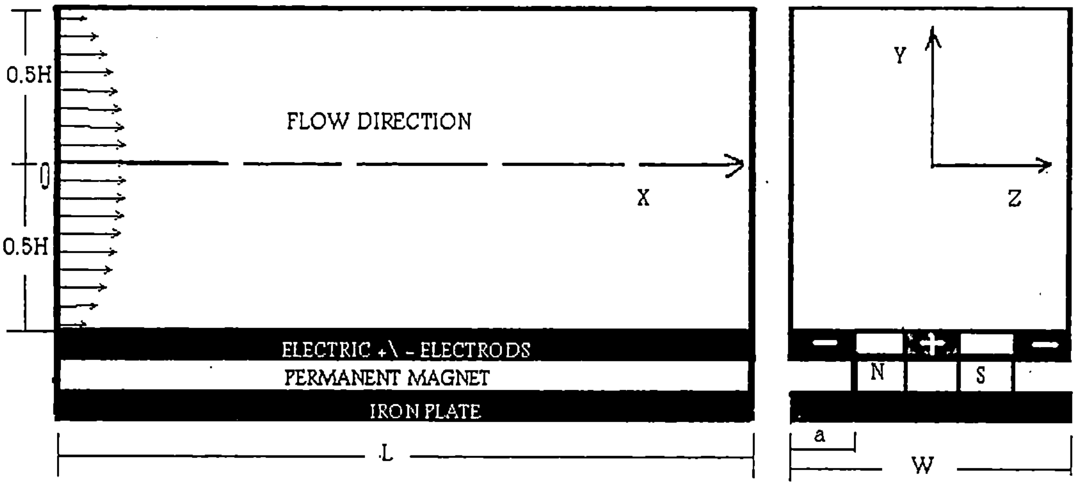

Fig. 2 Physical and coordinating systems

Where B_o is the magnetic field strength in Tesla, J_o is the electric current density in [A/m^2], a: is the electrode width and y is the vertical distance from the channel center. The third and fourth terms in the right hand side in momentum equation for x direction is source terms due to Lorentz force from lower and higher surfaces actuators. These forces are affected on x direction and decreases with y exponentially and independent of the flow velocity.

Let x be the direction of the flow , y is the direction normal to the flow and width of the plate parallel to Z direction, with assuming that the flow is two dimensional and fully developed flow for such configurations we have two cases.

3. Results and Conclusions

3.1First case: The force applied at the lower surface only:

From assumptions

$$u = u(y), v = 0 \quad \frac{\partial u}{\partial x} = 0$$

$$\therefore u \neq u(x) \qquad (3)$$

This means that the flow velocity u is independent of x and it is a function of y only.

From equation 3 we get that

$$\frac{\partial p}{\partial y} = 0 \qquad (4)$$

So the pressure drop depends only on x and this leads to:

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\frac{\partial^2 u}{\partial y^2} + \frac{\pi J_0 B_0}{8\rho}\exp\left[\frac{-\pi}{a}(y+h/2)\right] = 0 \quad (5)$$

$$u = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{y^2}{2} - \frac{a^2 J_0 B_0}{8\pi\mu}\exp\left[\frac{-\pi}{a}(y+h/2)\right] \quad (6)$$
$$+ Ay + B$$

3.1.1Boundary conditions:

At y= ±h/2, u=0

Substitute with the boundary conditions in equation (6) we get :

$$u=\frac{-h^2}{8\mu}\frac{\partial p}{\partial x}\left[1-4\left[\frac{y}{h}\right]^2\right]-\frac{a^2 J_o B_o}{16\pi\mu}\left[2e^{\frac{-\pi(y+\frac{h}{2})}{a}}+\frac{2y}{h}\left(1-e^{\frac{-\pi h}{a}}\right)-e^{\frac{-\pi h}{a}}-1\right] \quad (7)$$

This is a general equation for calculating velocity under combined effect of pressure gradient and the electromagnetic force.

To obtain the velocity under the effect of the electromagnetic force only, it can substitute in equation (7) with the value of pressure gradient equal to zero and in this case we get that:

$$u=-\frac{a^2 J_o B_o}{16\pi\mu}\left[2e^{\frac{-\pi(y+\frac{h}{2})}{a}}+\frac{2y}{h}\left(1-e^{\frac{-\pi h}{a}}\right)-e^{\frac{-\pi h}{a}}-1\right] \quad (8)$$

We define two characteristic velocities U_p; the velocity due to hydrodynamic pressure gradient and U_z the velocity due to electromagnetic body force and define by the Eq.9 and Eq.10 as following:

$$U_p=\frac{-h^2}{8\mu}\frac{\partial p}{\partial x} \quad (9)$$

$$U_z=\frac{a^2 J_o B_o}{16\pi\mu} \quad (10)$$

So equation (8) could take the forms as:

$$u = U_p\left[1-4\left[\frac{y}{h}\right]^2\right]- U_z\left[2e^{\frac{-\pi(y+\frac{h}{2})}{a}}+\frac{2y}{h}\left(1-e^{\frac{-\pi h}{a}}\right)-e^{\frac{-\pi h}{a}}-1\right] \quad (11)$$

For flow driven by electromagnetic force only:

$$u = -U_z\left[2e^{\frac{-\pi(y+\frac{h}{2})}{a}}+\frac{2y}{h}\left(1-e^{\frac{-\pi h}{a}}\right)-e^{\frac{-\pi h}{a}}-1\right] \quad (12)$$

Let us define, a P_{EM} This new dimensionless parameter; P_{EM} expresses the balance between hydrodynamic pressure forces to electromagnetic forces, which equal to:

The ratio between the two reference velocities Up/Uz is equal to

$$P_{EM}=\frac{U_p}{U_z} \quad (13)$$

The flow velocity equation can be transformed to the dimensionless form by divided the two sides by U_z it can be written as:

$$\frac{u}{U_z}=P_{EM}\left[1-4\left[\frac{y}{h}\right]^2\right]-\left[2e^{\frac{-\pi(y+\frac{h}{2})}{a}}+\frac{2y}{h}\left(1-e^{\frac{-\pi h}{a}}\right)-e^{\frac{-\pi h}{a}}-1\right] \quad (14)$$

The effect of P_{EM} on velocity variation at constant and different values for the ratio between the height and electrode width shown in fig.3

If the flow under the electromagnetic body force only, the electromagnetic parameter $P_{EM}=0$ and the Eq.14 has the form

$$\frac{u}{U_z}=-\left[2e^{\frac{-\pi(y+\frac{h}{2})}{a}}+\frac{2y}{h}\left(1-e^{\frac{-\pi h}{a}}\right)-e^{\frac{-\pi h}{a}}-1\right] \quad (15)$$

The electromagnetic parameter must be equal to zero, since $U_p=0$ due to vanishing the hydrodynamic pressure gradient.

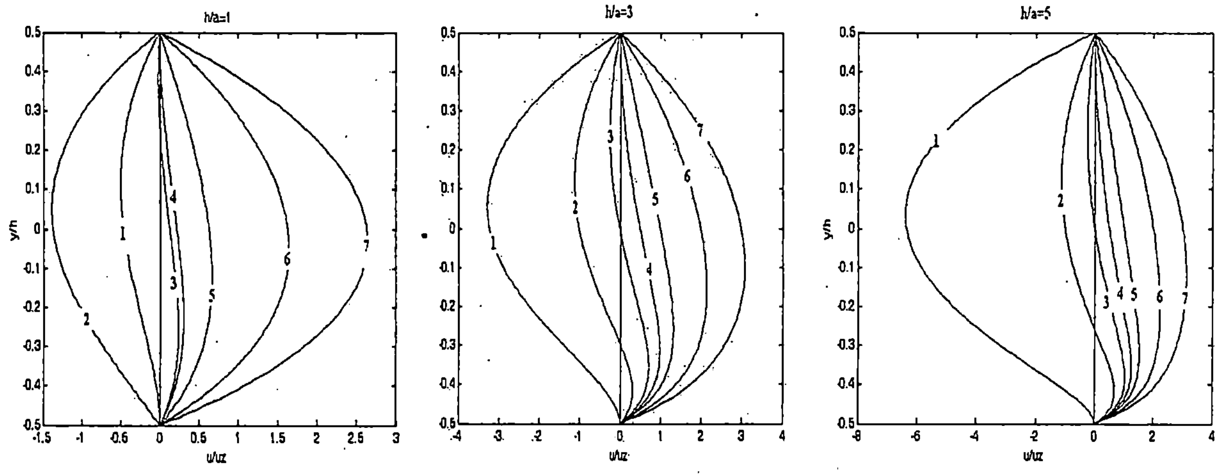

Fig.3 velocity distribution for 3 cases for h/a= (1, 3, 5) with different values for P_{EM}

Fig. 3 represents flow at h/a=1, 3, and 5 for 7 different P_{EM} values. In each case: Profile 1 illustrates flow with a critical P_{EM} values i.e. P_{EM} equal to: -1.0924033, -4.2124293 and -7.3539817 for h/a=1, h/a=3 and h/a=5 respectively at the lower surface where the shear stress equal to zero at this surface. We note that, the profile is normal on the upper surface at the point which meets this surface.

Profile 4 illustrates flow with a critical P_{EM} values i.e. P_{EM} equal to: -0.4105127, -0.499579 and -0.5 for h/a=1, h/a=3 and h/a=5 respectively at the upper surface where the shear stress equal to zero at this surface. We note that, the profile is normal on the upper surface at the point which meets this surface.

Profile 5 illustrates flow under electromagnetic force only i.e. P_{EM}=0, where the hydrodynamic pressure equal to zero and the flow pumping under the Lorentz force only

Profiles 2, 3, 6 and 7 represent general cases where the P_{EM} equal to -2, -1, 1 and 2 respectively which is not critical values. In all profiles; as the P_{EM} value increases; maximum velocity increases.

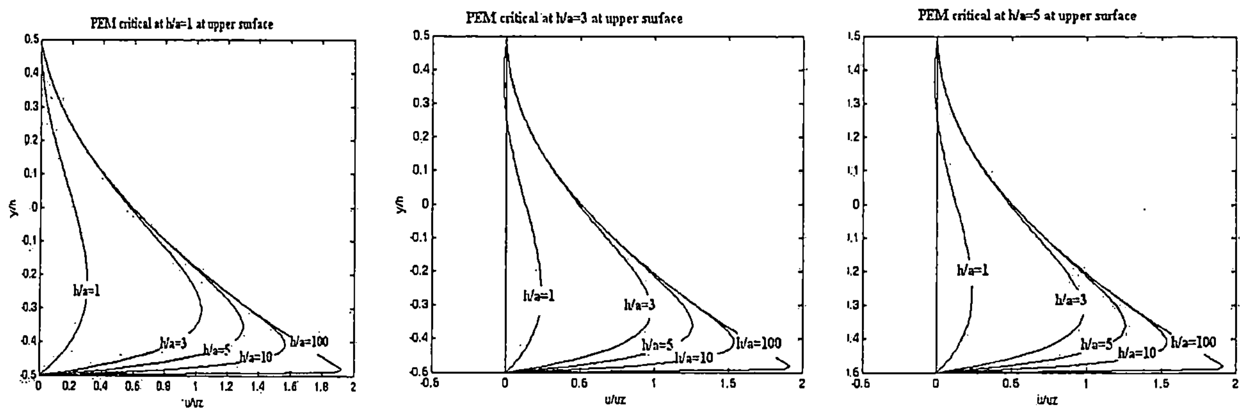

Fig.4 Lower plate has Lorentz force with different (h/a) values

Fig.4 represents flow velocity variation with the channel height, the first case all profiles at P_{EM} equal to -0.4105127 where it is the critical value at h/a=1. The second case all profiles at P_{EM} equal to -0.499579 where it is the critical value at h/a=3. The third case all profiles at P_{EM} equal to -0.5 where it is the critical value at h/a=5. The geometrical ratio is equal to 1, 3, 5, 10, and 100 in each case.

In each case, when the P_{EM} is corresponding to its critical value at the geometrical ratio h/a, the profile is vertical on the upper surface at the point which the profile intersect that surface.

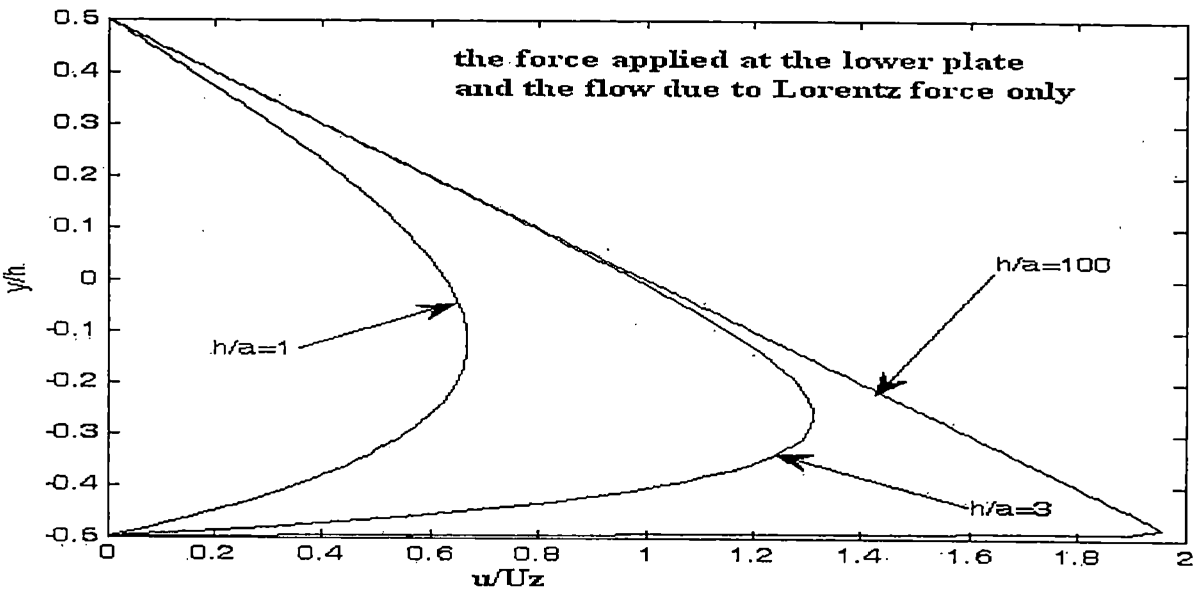

Fig. 5 flow due to the Lorentz force only, case of lower plate has force only

Fig.5 Flow due to Lorentz force only, and the force applied to the lower wall only, we note that as the ratio increase, the maximum velocity moving toward the lower surface. For the ratio equal to 100, the velocity profile approximately consists from two lines one is horizontal and the other is inclined with angle equal to 45°, its maximum value very near from the lower wall.

3.1.2Average Velocity:

The average velocity can be calculated as follows:

$$U_{av} = \frac{1}{h}\int_{\frac{-h}{2}}^{\frac{h}{2}} u\,\partial y = \frac{U_z}{h}\int_{\frac{-h}{2}}^{\frac{h}{2}} \frac{u}{U_z}\,\partial y \qquad (16)$$

$$U_{av} = \frac{U_z}{h}\int_{\frac{-h}{2}}^{\frac{h}{2}} \left[P_{EM}\left[1-4\left[\frac{y}{h}\right]^2\right] - \left[2e^{\frac{-\pi(y+\frac{h}{2})}{a}} + \frac{2y}{h}\left(1-e^{\frac{-\pi h}{a}}\right) - e^{\frac{-\pi h}{a}} - 1\right]\right]\partial y \qquad (17)$$

The average velocity can be also transformed to the dimensionless form by dividing the two sides by Uz and it can be written as follows:

$$\frac{U_{av}}{U_z}=\frac{Q}{hU_z}=\frac{2}{3}P_{EM}+\frac{2a}{\pi h}\left(e^{\frac{-\pi h}{a}}-1\right)+e^{\frac{-\pi h}{a}}+1 \qquad (18)$$

In case of no effect to electromagnetic force the pressure gradient equal to:

$$\frac{\partial p}{\partial x}=\frac{-12\mu}{h^2}[U_{av}] \qquad (19)$$

The pressure gradient in classical plane Poisuille's flow, also in case of the flow under the electromagnetic body force only where the hydrodynamic pressure gradient is vanishing and we get that:

$$\frac{\partial p}{\partial x}=\frac{12\mu U_z}{h^2}\left(\frac{2a}{\pi h}+1\right)\left(e^{\frac{-\pi h}{a}}+1\right) \qquad (20)$$

which represents the pressure gradient due to the electromagnetic force.

The dimensionless discharge Q and the dimensionless average velocity determined from Eq.18, the volumetric discharge per unit width of channel is equal to:

$$Q=hU_{av}=\left[\frac{2h}{3}U_p+\frac{hU_z}{2}\left(\frac{2a}{\pi h}+1\right)\left(e^{\frac{-\pi h}{a}}+1\right)\right] \qquad (21)$$

The discharge due to the electromagnetic force only can be obtained from:

$$Q=hU_{av}=\left[\frac{hU_z}{2}\left(\frac{2a}{\pi h}+1\right)\left(e^{\frac{-\pi h}{a}}+1\right)\right] \qquad (22)$$

3.1.3Shear stress:

From equation (11) of flow velocity with differentiation:

$$\frac{\partial u}{\partial y}=\left[-\frac{8y}{h^2}\right]U_p+U_z\left[\frac{2\pi}{a}e^{\frac{-\pi(y+\frac{h}{2})}{a}}-\frac{2}{h}\left(1-e^{\frac{-\pi h}{a}}\right)\right] \qquad (23)$$

The shear stress is obtained from:

$$\tau=-\mu\frac{\partial u}{\partial y}=\frac{8\mu y}{h^2}U_p-\mu U_z\left[\frac{2\pi}{a}e^{\frac{-\pi(y+\frac{h}{2})}{a}}-\frac{2}{h}\left(1-e^{\frac{-\pi h}{a}}\right)\right] \qquad (24)$$

For the flow under the electromagnetic force only the shear stress computed from:

$$\tau=-\mu U_z\left[\frac{2\pi}{a}e^{\frac{-\pi(y+\frac{h}{2})}{a}}-\frac{2}{h}\left(1-e^{\frac{-\pi h}{a}}\right)\right] \qquad (25)$$

Taking the original value of shear stress equal to:

$$\tau_o=-\mu\frac{U_z}{h} \qquad (26)$$

So the dimensionless shear stress could be determined from:

$$\frac{\tau}{\tau_o}=\frac{8y}{h}P_{EM}+\frac{2\pi h}{a}e^{\frac{-\pi(y+\frac{h}{2})}{a}}-2\left(1-e^{\frac{-\pi h}{a}}\right) \qquad (27)$$

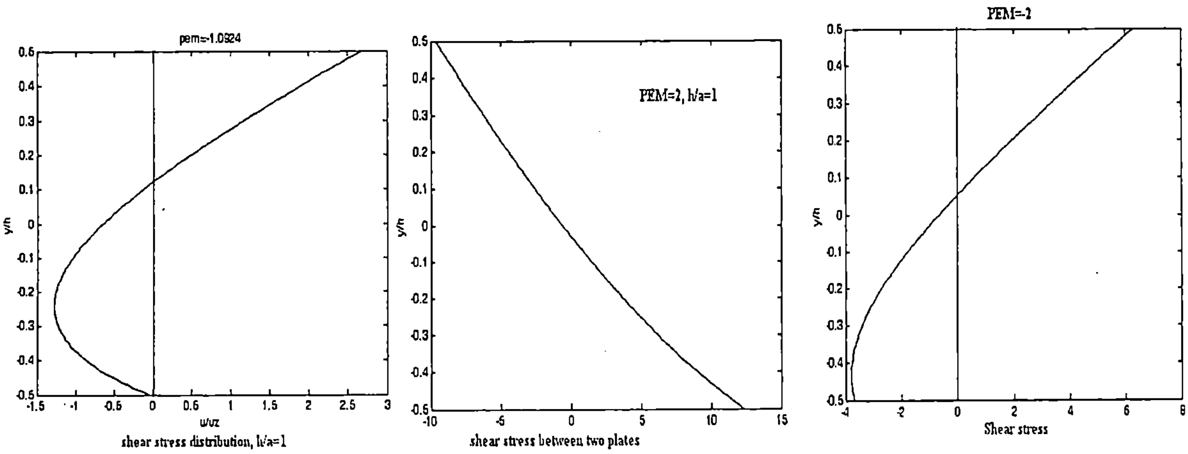

Fig. 6 variation of shear stress with channel height at different P_{EM} cases

Fig.6 illustrates the shear stress distribution with the channel height first case, the shear stress equal to zero at the lower plate, since the value of electromagnetic parameter is a critical value at h/a=1, the reminder cases are a general shear stress distribution for P_{EM}=2 and P_{EM}=-2. The 3 cases at h/a=1. when P_{EM} is negative value; shear stress increase with channel height and when it is equal to a positive value the shear stress increases as the height decreases.

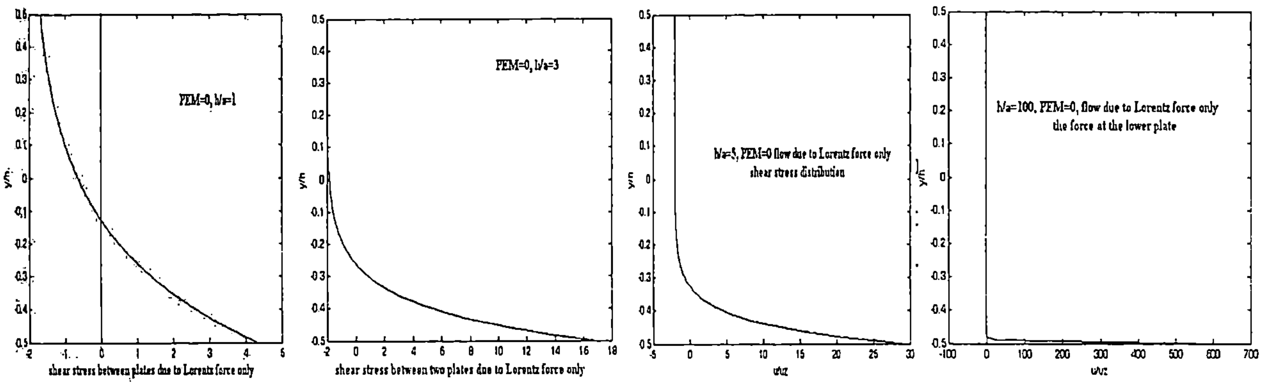

Fig.7 illustrates the variation of shear stress for four cases of h/a= (1, 3, 5 and 100) due to Lorentz force only

In all cases the shear stress decreases as the height of channel increases.

To find the shear stress at the upper and lower plates, put y= ±h/2 so, for the combined effect: The value of shear stress at the upper and lower plates is calculated from:

$$\because \tau_{y=\frac{-h}{2}} = -\mu\frac{\partial u}{\partial y} = \frac{-4\mu}{h}U - \mu U_z\left[\frac{2\pi}{a} - \frac{2}{h}\left(1-e^{\frac{-\pi h}{a}}\right)\right] \quad (28\text{-a})$$

$$\therefore \tau_{y=\frac{h}{2}} = -\mu\frac{\partial u}{\partial y} = \frac{\mu}{h}\left[4U_p - U_z\left(\frac{2\pi h}{a}e^{\frac{-\pi h}{a}} - 2\left(1-e^{\frac{-\pi h}{a}}\right)\right)\right] \quad (28\text{-b})$$

For the flow under the electromagnetic force only the Eq., (28-a) and (28-b) can be written as

$$\therefore \tau_{y=-\frac{h}{2}} = -\mu\frac{\partial u}{\partial y} = \frac{\mu U_z}{h}\left(\frac{2\pi h}{a} - 2\left(1-e^{\frac{-\pi h}{a}}\right)\right) \quad (29\text{-a})$$

$$\therefore \tau_{y=\frac{h}{2}} = -\mu\frac{\partial u}{\partial y} = \frac{\mu U_z}{h}\left(\frac{2\pi h}{a} - 2\left(1-e^{\frac{-\pi h}{a}}\right)\right) \quad (29\text{-b})$$

3.1.4The condition for vanishing shear stress at lower and upper plates:

For zero shear stress the lift hand side of equation (24) must be vanish

$$\text{so: } \tau = \frac{8\mu y}{h^2}U_p - \mu U_z\left[\frac{2\pi}{a}e^{\frac{-\pi(y+\frac{h}{2})}{a}} - \frac{2}{h}\left(1-e^{\frac{-\pi h}{a}}\right)\right] = 0$$

This leads to

$$\frac{U_p}{U_z} = P_{EM} = \frac{h}{8y}\left[2\pi\left[\frac{h}{a}\right]e^{\frac{-\pi(y+\frac{h}{2})}{a}} - 2\left(1-e^{\frac{-\pi h}{a}}\right)\right] \quad (30)$$

for zero shear stress at the lower and upper plates the electromagnetic parameter P_{EM} take the next values after putting y=±h/2 respectively.

$$P_{EM_{y=-h/2}} = \frac{-1}{2}\left[\frac{\pi h}{a} + e^{\frac{-\pi h}{a}} - 1\right] \quad (31)$$

$$P_{EM_{y=h/2}} = \frac{1}{2}\left[e^{\frac{-\pi h}{a}}\left(\frac{\pi h}{a}+1\right) - 1\right] \quad (32)$$

We note that the value of electromagnetic parameter required for vanishing shear stress at the walls is a function on the ratio between the height h and the electrode width a.

For the flow under the electromagnetic force only, P_{EM} must vanish since the velocity due to hydrodynamic pressure is vanishing. This gives a unique point depending on the ratio between the channel heights to the electrode width that happened in this flow which the shear stress is vanishing. So we find that the position for no shear stress under the electromagnetic force only depending on the geometry of the problem i.e. the ratio between the channel height and the width of the electrode only and is independent on the flow characteristics.

The following table gives the values of critical electromagnetic parameter at the lower and upper plates that required for zero shear stress.

h/a	PEM at y=-h/2	PEM at =h/2
0.1	-0.0222809	-0.0200673
0.5	-0.3893379	-0.1632683
1	-1.0924033	-0.4105127
3	-4.2124293	-0.499579
5	-7.3539817	-0.5
10	-15.207963	-0.5
100	-156.57963	-0.5

3.1.5 The skin friction coefficient can be defined as:

$$\therefore C_f = \frac{\tau}{\frac{1}{2}\rho U_{av}^2} \quad (33)$$

Substituting with the values of shear stress from Eq., (27) and average velocity from Eq., (14) the skin friction coefficient can be written as:

$$\therefore C_f = \frac{\frac{4y}{h}P_{EM} - \frac{\pi h}{a}e^{\frac{-\pi(y+\frac{h}{2})}{a}} + 1}{Re\left[\frac{1}{6}P_{EM} + \frac{a}{2\pi h}(e^{\frac{-\pi h}{a}} - 1) + 0.25(e^{\frac{-\pi h}{a}} + 1)\right]} \quad (34)$$

The skin friction coefficient at lower and upper plates can be obtained as:

$$\therefore C_{f_{y=-h/2}} = \frac{-2P_{EM} - \frac{\pi h}{a} - e^{\frac{-\pi h}{a}} + 1}{Re\left[\frac{1}{6}P_{EM} + \frac{a}{2\pi h}(e^{\frac{-\pi h}{a}} - 1) + 0.25(e^{\frac{-\pi h}{a}} + 1)\right]} \quad (35\text{-a})$$

$$\therefore C_{f_{y=h/2}} = \frac{2P_{EM} - \left(\frac{\pi h}{a}+1\right)e^{\frac{-\pi h}{a}} + 1}{Re\left[\frac{1}{6}P_{EM} + \frac{a}{2\pi h}(e^{\frac{-\pi h}{a}} - 1) + 0.25(e^{\frac{-\pi h}{a}} + 1)\right]} \quad (35\text{-b})$$

For the flow under electromagnetic force only, Eq., (35-a) and Eq., (35-b) can be written as:

$$C_f = \frac{-\frac{\pi h}{a}e^{\frac{-\pi(y+\frac{h}{2})}{a}} - e^{\frac{-\pi h}{a}} + 1}{Re\left[\frac{a}{2\pi h}(e^{\frac{-\pi h}{a}} - 1) + 0.25(e^{\frac{-\pi h}{a}} + 1)\right]} \quad (36)$$

At the lower plates its value is determine from:

$$\therefore C_{f_{y=-h/2}} = \frac{-\frac{\pi h}{a} - e^{\frac{-\pi h}{a}} + 1}{Re\left[\frac{a}{2\pi h}(e^{\frac{-\pi h}{a}} - 1) + 0.25(e^{\frac{-\pi h}{a}} + 1)\right]} \quad (37\text{-a})$$

And at the upper plate its value is determine from:

$$\therefore C_{f_{y=h/2}} = \frac{-\left(\frac{\pi h}{a}+1\right)e^{\frac{-\pi h}{a}} + 1}{Re\left[\frac{a}{2\pi h}(e^{\frac{-\pi h}{a}} - 1) + 0.25(e^{\frac{-\pi h}{a}} + 1)\right]} \quad (37\text{-b})$$

3.2 Case 2: Upper and Lower Plates have Electromagnetic Forces:

Here the exponentially decayed electromagnetic body force is applied on the lower and upper channel's plates and the force is exponentially decayed from the position of its effect.

$$u=\frac{1}{\mu}\frac{\partial p}{\partial x}\frac{y^2}{2}-\frac{a^2 J_o B_o}{8\pi\mu}e^{\left(\frac{-\pi}{a}(y+h/2)\right)}-\frac{a^2 J_o B_o}{8\pi\mu}e^{\left(\frac{\pi}{a}(y-h/2)\right)}+Ay+B \qquad (38)$$

3.2.1 Boundary conditions

y = -h/2, u=0 and y = h/2, u=0

Substitute in Eq.9 we get that A = zero and B equal to:

$$B=-\frac{1}{\mu}\frac{\partial p}{\partial x}\frac{h^2}{4}+\frac{a^2 J_o B_o}{8\pi\mu}\left(1+e^{\frac{-\pi h}{a}}\right) \qquad (39)$$

3.2.2 Flow velocity

Substitute with values of A and B in Eq. 38

$$u=\frac{-1}{\mu}\frac{\partial p}{\partial x}\frac{h^2}{8}\left(1-4\left(\frac{y}{h}\right)^2\right)-\frac{a^2 J_o B_o}{8\pi\mu}\left(e^{\left(\frac{-\pi}{a}(y+h/2)\right)}+e^{\left(\frac{\pi}{a}(y-h/2)\right)}-\left(1+e^{\frac{-\pi h}{a}}\right)\right) \qquad (40)$$

As before the hydrodynamic flow velocity is equal to: $u_p=\frac{-1}{\mu}\frac{\partial p}{\partial x}\frac{h^2}{8}$

And the flow velocity due to the electromagnetic forces is equal to: $u_z=\frac{a^2 J_o B_o}{16\pi\mu}$

$$\text{So, } u=u_p\left(1-4\left(\frac{y}{h}\right)^2\right)-2u_z\left(e^{\left(\frac{-\pi}{a}(y+h/2)\right)}+e^{\left(\frac{\pi}{a}(y-h/2)\right)}-\left(1+e^{\frac{-\pi h}{a}}\right)\right) \qquad (41)$$

Divide the two sides by U_z we get that:

$$\frac{u}{u_z}=P_{EM}\left(1-4\left(\frac{y}{h}\right)^2\right)-2\left(e^{\left(\frac{-\pi}{a}(y+h/2)\right)}+e^{\left(\frac{\pi}{a}(y-h/2)\right)}-\left(1+e^{\frac{-\pi h}{a}}\right)\right) \qquad (42)$$

If the flow under the electromagnetic force only so the hydrodynamic pressure equals to zero and we get that:

$$\frac{u}{u_z}=2\left(1+e^{\frac{-\pi h}{a}}-e^{\left(\frac{-\pi}{a}(y+h/2)\right)}-e^{\left(\frac{\pi}{a}(y-h/2)\right)}\right) \qquad (43)$$

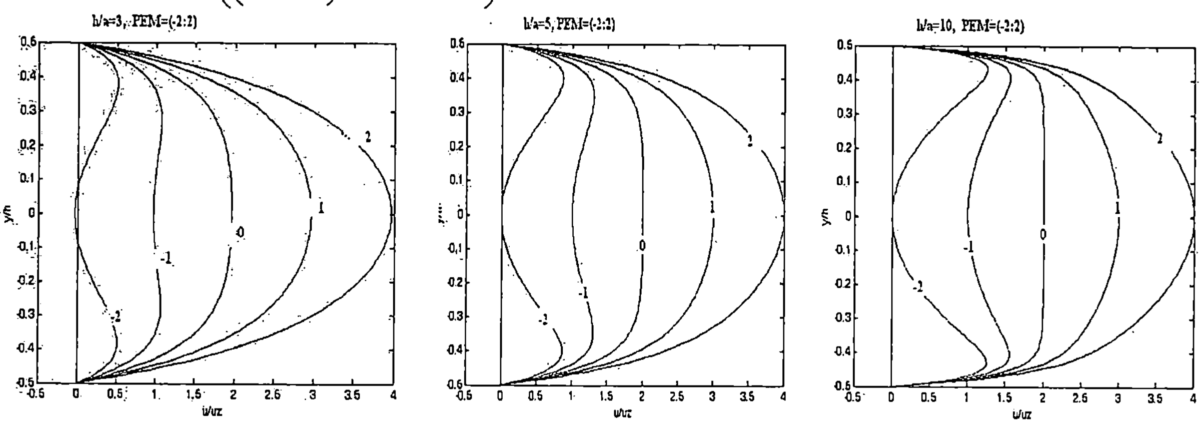

Fig.8 the velocity profiles at constant h/a and P_{EM} = (-2:2)

Fig.8 illustrates three cases for flow at h/a=3, 5 and 10 at different PEM values from (-2:2), the profile which represent the flow due to electromagnetic force only, is the profile drawn at PEM equal to zero.

The maximum velocity is increasing as the geometrical ratio is increasing, and the maximum velocity appears beside the wall more and more as the ratio increases. M shape profiles appear at the negative values of PEM.

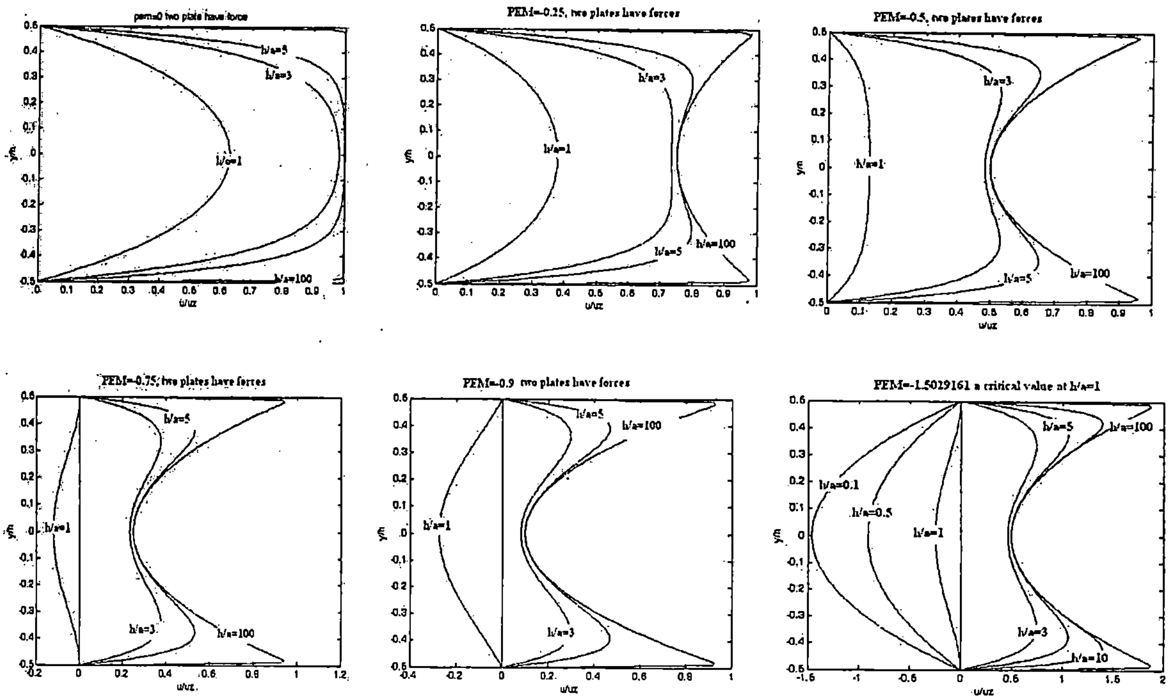

Fig.9 different cases for velocity profiles at constant P_{EM} and h/a= 1, 3, 5 and 100

Fig.9 represents 6 cases for the relation between the flow velocity and height of channel each case drawn at constant Electromagnetic parameter as shown each case drawn at four geometrical ratio h/a. first case, illustrates the flow at PEM equal to zero i.e. the flow due to electromagnetic force only. As the geometrical ratio increases the velocity increases and the maxim velocity fill more regions between the two plates, and the boundary layer where the velocity is varying be compacted more and more, when h/a reaches 100 the boundary layer is so compacted and maximum velocity appearing besides the walls and filling all the regions between the two plates. For all cases as the geometrical ratio and PEM increase the maximum velocity increases. Also as PEM is reaching larger values, M shape velocity profiles appear and the velocity beside the walls is larger than its values at the center of distance between the plates. In the last case the shear stress equal to zero at h/a=1 since the PEM=-1.5029161 which it is a critical value at this ratio.

3.2.3 Dimensionless flowrate:

The dimensionless average velocity and dimensionless flow rate could be determined as follow;

$$\frac{Q}{hU_z} = \frac{U_{av}}{U_z} = \frac{1}{h}\int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{u}{U_z}\,\partial y \qquad (44)$$

From the relation for the dimensionless flow velocity equation (42)

$$\frac{U_{av}}{U_z} = \frac{Q}{hU_z} = \frac{1}{h}\int_{-\frac{h}{2}}^{\frac{h}{2}} u\,\partial y = \frac{1}{h}\int_{-\frac{h}{2}}^{\frac{h}{2}} \left(P_{EM}\left(1-4\left(\frac{y}{h}\right)^2\right) - 2\left(e^{\left(\frac{-\pi}{a}\left(y+\frac{h}{2}\right)\right)} + e^{\left(\frac{\pi}{a}\left(y-\frac{h}{2}\right)\right)} - \left(1+e^{\frac{-\pi h}{a}}\right)\right)\right)\partial y \qquad (45)$$

By integration we get that:

$$\frac{U_{av}}{U_z} = \frac{Q}{hU_z} = \frac{2P_{EM}}{3} + \frac{4a}{\pi h}\left[e^{-\frac{\pi h}{a}} - 1\right] + 2\left[e^{-\frac{\pi h}{a}} + 1\right] \qquad (46)$$

3.2.4 Wall shear stress and skin friction coefficient:

For the laminar flow range the shear stress determined from:

$$\tau = -\mu\frac{\partial u}{\partial y} = -\mu U_z \frac{\partial}{\partial y}\frac{u}{U_z} \qquad (47)$$

The dimensionless velocity gradient is determined from the equation (42) for velocity distribution by differentiation with respected to the Y axis as:

$$\frac{\partial}{\partial y}\frac{u}{U_z} = -8\frac{y}{h^2}P_{EM} + \frac{2\pi}{a}e^{-\frac{\pi}{a}\left(y+\frac{h}{2}\right)} - \frac{2\pi}{a}e^{\frac{\pi}{a}\left(y-\frac{h}{2}\right)} \qquad (48)$$

So the shear stress in dimensionless form could be calculated from:

$$\frac{\tau}{\tau_o} = -8\frac{y}{h}P_{EM} + \frac{2\pi h}{a}e^{-\frac{\pi}{a}\left(y+\frac{h}{2}\right)} - \frac{2\pi h}{a}e^{\frac{\pi}{a}\left(y-\frac{h}{2}\right)} \qquad (49)$$

Where $\tau_o = -\frac{\mu U_z}{h}$

3.2.5 Condition for vanishing shear stress:

By putting the value of shear stress equal to zero in equation (49) we get that:

$$P_{EM} = \frac{\pi h^2}{4ay}\left[e^{-\frac{\pi}{a}\left(y+\frac{h}{2}\right)} - e^{\frac{\pi}{a}\left(y-\frac{h}{2}\right)}\right] \qquad (50)$$

So, we find that the value of electromagnetic parameter required for vanishing shear stress is depending on the position of calculations y/h and the geometrical channel ratio between the height and electrode width h/a.

Also, the shear stress at the lower and upper walls is vanished if:

$$P_{EM} = -\frac{\pi h}{2a}\left[1 - e^{-\frac{\pi h}{a}}\right] \qquad (51)$$

by substituting by y=h/2 and –h/2 and we note that the value of the electromagnetic parameter required for zero shear stress at the two walls in same time and depending only on the channel geometrical ratio h/a. and its values changed with h/a as given in following table:

h/a	P_{EM}
0.1	-0.0423482
0.5	-0.6221299
1	-1.5029161
3	-4.7120087
5	-7.8539805
10	-15.707963
100	-157.07963

3.2.6 Skin friction coefficient C_f:

For the range of laminar flow the skin friction coefficient could be calculated from the following relation:

$$C_f = \frac{\tau}{\frac{1}{2}\rho U_{av}^2}$$

Substituting by the values of average velocity and shear stress we could deduce the following relation:

$$C_f = \frac{4\left[\frac{y}{h}\right]P_{EM} - \frac{\pi h}{a} e^{-\frac{\pi}{a}(y+h/2)} + \frac{\pi h}{a} e^{\frac{\pi}{a}(y-h/2)} + 1}{Re\left[\frac{P_{EM}}{6} + \frac{a}{\pi h}\left(e^{-\frac{\pi h}{a}} - 1\right) + 0.5\left(e^{-\frac{\pi h}{a}} + 1\right)\right]} \qquad (52)$$

And for the flow under the electromagnetic force only the skin friction could be calculated from same equation putting the value of electromagnetic parameter equal to zero and the relation takes the form:

$$C_f = \frac{-\frac{\pi h}{a} e^{-\frac{\pi}{a}(y+h/2)} + \frac{\pi h}{a} e^{\frac{\pi}{a}(y-h/2)} + 1}{Re\left[\frac{a}{\pi h}\left(e^{-\frac{\pi h}{a}} - 1\right) + 0.5\left(e^{-\frac{\pi h}{a}} + 1\right)\right]} \qquad (53)$$

4. Conclusions

The influence of a streamwise Lorentz force on the flow along the surface has been studied. That show a strong acceleration of the near wall flow if the electromagnetic forces of sufficient strength are applied. The application of the streamwise force to control of separation has been successfully demonstrated. Separation is delayed resulting in a momentum increase in the near wall regions flow, this overcomes the momentum deficit in boundary layer and the total drag is decreasing as a result.

The flowrate and the average velocity values are increasing under the effect of the electromagnetic force; also the electromagnetic Lorentz force in streamwise direction able to derive the flow in the absence of hydrodynamic pressure gradient.

The position of the maximum flow velocity is controlled by the value of electromagnetic parameter, since it towards to the wall with varying the parameter value. If the flow moving without any hydrodynamic pressure gradient; the resulting flow under the electromagnetic streamwise Lorentz force only modifying the velocity distribution. And when the ratio between channel height and electrode width is equal to 100; the velocity profile consider as a two straight line; one is horizontal and the other is inclined with 45° and has the maximum velocity value near the wall regions.

The shear stress at the wall was controlled and for a certain values of electromagnetic parameter, it could be totally delayed at the surface.

In case of two plates have forces; the shear stress is totally delayed at the same time for a certain values of electromagnetic parameter. These values are changed with changing of the ratio between the height and electrode width; in this case the flow moving as a bulky flow i.e. the value of shear stress is equal to zero. This leads to decreasing the Friction and drag.

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