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Critical Buckling Load of Laced Columns.

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Critical Buckling Load of Laced Columns
" حمل الانبعاج الحرج للأعمدة ذات الأربطة "

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الخلاصة:
إن الهدف الرئيسي لهذا البحث هو دراسة تأثير طريقة ترتيب وأبعاد عناصر الربط على القيم الفعلية لحمل الانبعاج الحرج للأعمدة ذات الأربطة والتي تتكون من أربعة أرجل وذلك للوصول للأمثل منها لاستخدامها في تصميم العناصر الرئيسية الهامة في تغطية الاستادات الرياضية حيث تمت دراسة أربع أشكال لطرق ترتيب هذه العناصر مع نوعين من قيود النهايات للأعمدة آخذا في الاعتبار تأثير ألواح الربط عند حساب حمل الانبعاج الحرج للأعمدة وقد تم استخدام برنامج حاسب آلي قائم على طريقة تصغير طاقة الوضع باستخدام طريقة المنحدرات المتبادلة وأيضا طريقة منحنى الصلابة علما بأن نتائج التحليل باستخدام هذه الطريقة قد أظهرت توافقا كبيرا مع تلك التي تم الحصول عليها باستخدام برنامج ساب2000.

Abstract
The aim of the present work is to investigate the effect of the detailed arrangement and dimensions of the lacing bars on the actual value of the critical buckling load of a Laced Column having four legs. The critical buckling load is calculated in order to reach for the best arrangement of lacing systems to use them as a one main structural element and main component for skeleton used for covering a long span systems. The study is carried out for four different forms of lacing arrangement and two different forms of restraints taking the effect of inertia of the gusset plate for a laced column into account when calculating its critical buckling load. A computer program constructed by [1] which is based upon the minimizing of total potential energy by the conjugate gradient technique and the method of Stiffness Curve created by the author is used for this study. This good preliminary technique is examined using Sap2000 program and the results are in a good agreement.

Key words
Laced Column, Critical Buckling Load, Gusset Plate, Euler Load

1. Introduction
The behavior of Laced columns is different from solid columns due to the effect of shear deformations. This effect on the elastic critical load of columns was studied for the first time by Engesser [2], who used the modified slenderness method to consider this effect. Bleich [3], Timoshenko and Gere [4] and several other researchers [5-9] also studied the effect of shear deformations on the elastic critical load of built-up columns, either theoretically or experimentally.
The study is carried out for four different forms of lacing arrangements and two different forms of end conditions taking the effect of inertia of the gusset plate for a laced column into account when calculating its critical buckling load. This study is carried out by considering an increase in the inertia of the part of member fastened by gusset plate and taken as 0.05, 0.1 and 0.15 of the total length of the member.

2. Effect of Shear Deformation on The Critical Buckling Load of a Laced Column
The critical load for a laced column is always less than for a solid column having

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the same cross-sectional area and the same slenderness ratio *l/r*. This decrease in the critical load is due primarily to the fact that the effect of shear on deflections is much greater for a laced column than for a solid bar. The actual value of the critical load depends upon the detailed arrangement and dimensions of the lacing bars.

The critical load can be obtained using the following equation:

$$P_{cr} = \frac{P_e}{1 + P_e/P_d} \qquad (1)$$

where P_e is the Euler critical load and the factor $1/P_d$ is the quantity by which the shearing force is multiplied in order to obtain the additional slope of the deflection curve due to shear. Thus we have:

$$P_{cr} = K.P_e \qquad (2)$$

Where $P_e = \frac{\pi^2 EI}{L^2}$ for pinned-ended column

$P_e = \frac{\pi^2 EI}{4L^2}$ for fixed-free column

From Eq. (1) and Eq. (2):

$$K = \frac{1}{1 + P_e/P_d} \qquad (3)$$

Where K is a factor depends on the detailed arrangement and dimensions of the lacing bars.

If the cross-sectional areas of the lacing elements are small in comparison with the area of the main members, the critical load given by Eq. (1) may be considerably lower than the Euler value. Thus the laced column may be considerably weaker than a solid strut with the same EI, but since the amount of material used is less, the laced column may be more economical.

A FORTRAN program to generate data for the columns is constructed by the authors. Then, another Fortran program constructed by [1] is used to calculate the critical buckling load using the method of the stiffness curve and checked by the Sap2000 program that provides big facilities for building the models besides efficiency, high speed in performing such analysis and displaying results. The analysis is carried out using the Fortran program by taking $P - \Delta$ effect for calculating the initial displacement due to a lateral disturbing force Q equals to $(0.02P_e)$. Then, the analysis is performed at the end of the previous analysis starting with p equals to, for example, $0.1P_e$ and continuing to increase the value of p and calculating the corresponding stiffness for the column until the stiffness of the structure becomes zero. Then the analysis stops and the critical load equals to the load parameter at this case. The results are compared using the Sap2000 program in which the analysis begins also with a static nonlinear case taking $P - \Delta$ effect for calculating the initial displacement due to the lateral disturbing force equals to $(Q = 0.02P_e)$. After that, we carry out the buckling analysis at the end of the nonlinear case. The comparison between some results from both methods is shown later in Fig.(18) and Table (8).

2.1. Configuration of Columns

Two different end restrained columns are studied, one of them is Fixed-Free and the other is Pinned-Ended Column. Both columns have four different bracing arrangements. The properties of columns are given in Table (1). The arrangements for lacing bars for both columns are shown in Fig. (1) and Fig. (2).

Table (1) Column Properties and dimensions

IC = 8.32x10-7 m^4	AC =1.178x10-3 m^2
B = 1.0 m	H = 10.0 m
E = 2100 t/cm^2	U = 0.3

2.2. Pinned-Ended Column

The Euler load for the columns is calculated as follows considering them as solid columns:

$$P_e = \frac{\pi^2 E(IC + AC(B/2)^2)(4)}{10^2} = 2448.4\ t$$

Then, the actual critical load and the factor K for the columns taking effect of the bracing arrangement and properties using SAP2000 are computed. The effect of increasing and decreasing the inertia and area of all element components for the pinned-ended column on the critical load and the factor K is carried out with reference to Table (2) and (3) which have all properties taken into consideration.

The obtained results showed that the increase of the moment of inertia and area of the column leads to the increase of the Euler and critical load with the same ratio. The factor K stills constant for a given value of IB/IC (the ratio of the moment of inertia of the bracing members to that of the vertical members). Also the ratio of IB/IC which makes the total weight of the column small enough to give a large buckling critical load is (0.5). See Fig. (3), (4) and (5). The buckling shapes of the columns are shown in Fig. (6).

2.3. Fixed-Free Column

The Euler load for the columns is calculated also as follows considering them as solid columns:

$$P_e = \frac{\pi^2 E(IC + AC(B/2)^2)(4)}{4x10^2} = 612.11\ t$$

Then, the effect of the bracing arrangement and properties on the actual critical load and thus the factor K for Fixed-Free the columns is also carried out. See Table (4) and (5).

The obtained results showed that the increase of the moment of inertia and area of the column leads to the increase of the Euler and critical load with the same ratio. The factor K stills constant for a given value of IB/IC (the ratio of the moment of inertia of the bracing members to that of the vertical members). Also the ratio of IB/IC which makes the total weight of the column small enough to give a large buckling critical load is (0.5). See Fig. (7), (8) and (9). The buckling shapes of the columns are shown in Fig. (10).

3. Effect of Gusset Plate on The Critical Buckling Load of a Laced Column

The effect of gusset plate on the critical buckling load and the factor K is carried out for columns having $IB/IC = 1/6$ .The length of member fastened by gusset plate (Per) is taken 0.05, 0.1 and 0.15 from the total length of the member as shown in Fig. (11). The inertia of the part of member fastened by gusset plate is increased as shown in Fig. (12) and (13) and the analysis is carried out for three different values of the thickness of gusset plate (8 - 10 - 12 mm).

3.1. Pinned-Ended Column

From the results shown in Table (6) we noticed that taking the effect of gusset plate into account increases the critical buckling load by approximately 26 - 33 % for 0.15L and by 13 - 18 % for 0.1L and by 3 - 7 % for 0.05L from its value. Also, it increases the critical load of columns having $IB/IC = 1/6$ to that having $IB/IC = 1$ as shown in Fig. (14) and (15). The obtained results also showed that increasing the thickness of gusset plate increases the critical buckling load with minimal degree.

3.2. Fixed-Free Column

From the results shown in Table (7) we noticed that taking the effect of gusset plate into account increases the critical buckling load by approximately 22 - 32 % for 0.15L and by 11- 17 % for 0.1L and by 3 - 6 % for 0.05L from its value. Also it increases the critical load of columns having $IB/IC = 1/6$ to that having $IB/IC = 1$ as shown in Fig. (16) and (17).

4. Conclusion

In this study, 8-laced type, built-up columns with various features were analyzed to obtain their critical buckling load and the following points can be concluded:

I. The laced column may be considerably weaker than a solid strut with the same EI, but the light material improve the results and the laced column may be more economical.

II. Increasing both moment of inertia and area of the column leads to an increase of the Euler and critical load with the same ratio. The factor K stills constant for a given value of IB/IC (the ratio of the moment of inertia of the bracing members to that of the vertical members).

III. The ratio of IB/IC of (0.5) makes the total weight of the column small enough to give a large buckling critical load and close to the Euler load.

IV. Taking the effect of gusset plate into account in the analysis increases the critical buckling load by 22 - 33 % for gusset having 0.15L, in which L represents the members' length. The increase of critical load with gussets having 0.1L and 0.05L is 11 - 18 % and 3 - 7 % respectively.

V. Taking the effect of gusset plate increases the critical load of columns having $IB/IC = 1/6$ to that having $IB/IC = 1$. Also, increasing the thickness of gusset plate increases the critical buckling load with minimal degree.

5. List of symbols

AC The cross sectional area
B The column width (square)
E Modulus of elasticity
H The column height
IB Moment of inertia of the horizontal and diagonal members (bracing)
IC Moment of inertia of vertical member
IP The type of lacing bars arrangement
K Critical buckling load / Euler load
LG Length of gusset plate
l/r Slenderness ratio
Per The length of member fastened by gusset plate
P_{cr} Critical buckling load
P_e Euler load
Q Disturbing force
TG Thickness of gusset plate
U Poisson's ratio

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Table (2) Effect of increasing properties of Pinned-Ended column on P_{cr} and K for $IP = 1$

IC m^4	AC m^2	IP	P_e ton	IB/IC	P_{cr} ton	K
I	A	1	2448.44	1	2162.54	0.88
I	A	1	2448.44	1/2	1998.78	0.82
I	A	1	2448.44	1/3	1847.86	0.75
I	A	1	2448.44	1/4	1715.44	0.70
I	A	1	2448.44	1/5	1601.63	0.65
I	A	1	2448.44	1/6	1507.19	0.62
2I	2A	1	4896.88	1	4325.07	0.88
2I	2A	1	4896.88	1/2	3997.57	0.82
2I	2A	1	4896.88	1/3	3695.72	0.75
2I	2A	1	4896.88	1/4	3430.87	0.70
2I	2A	1	4896.88	1/5	3203.25	0.65
2I	2A	1	4896.88	1/6	3014.38	0.62
3I	3A	1	7345.32	1	6487.61	0.88
3I	3A	1	7345.32	1/2	5996.35	0.82
3I	3A	1	7345.32	1/3	5543.58	0.75
3I	3A	1	7345.32	1/4	5146.31	0.70
3I	3A	1	7345.32	1/5	4804.88	0.65
3I	3A	1	7345.32	1/6	4521.57	0.62

Table (3) Effect of changing the arrangement of the lacing bars and decreasing its dimensions on P_{cr} and K for Pinned-Ended columns with $IP = 1:4$

IC m^4	AC m^2	IP	P_e ton	IB/IC	P_{cr} ton	k	Total weight ton
I	A	1	2448.44	1	2162.54	0.88	0.9160
I	A	1	2448.44	1/2	1998.78	0.82	0.6510
I	A	1	2448.44	1/3	1847.86	0.75	0.5630
I	A	1	2448.44	1/4	1715.44	0.70	0.5190
I	A	1	2448.44	1/5	1601.63	0.65	0.4920
I	A	1	2448.44	1/6	1507.19	0.62	0.4750
I	A	2	2448.44	1	2633.43	1.08	1.5560
I	A	2	2448.44	1/2	2328.21	0.95	0.9710
I	A	2	2448.44	1/3	2171.40	0.89	0.7760
I	A	2	2448.44	1/4	2057.21	0.84	0.6790
I	A	2	2448.44	1/5	1960.37	0.80	0.6200
I	A	2	2448.44	1/6	1874.47	0.77	0.5810
I	A	3	2448.44	1	2095.07	0.86	1.1380
I	A	3	2448.44	1/2	1923.67	0.79	0.7620
I	A	3	2448.44	1/3	1793.43	0.73	0.6370
I	A	3	2448.44	1/4	1660.73	0.68	0.5740
I	A	3	2448.44	1/5	1545.76	0.63	0.5370
I	A	3	2448.44	1/6	1445.67	0.59	0.5120
I	A	4	2448.44	1	2220.11	0.91	1.3810
I	A	4	2448.44	1/2	1999.08	0.82	0.8840
I	A	4	2448.44	1/3	1827.73	0.75	0.7180
I	A	4	2448.44	1/4	1678.89	0.69	0.6350
I	A	4	2448.44	1/5	1547.78	0.63	0.5850
I	A	4	2448.44	1/6	1434.08	0.59	0.5520

Table (4) Effect of increasing properties of Fixed-Free column on P_{cr} and K for $IP = 1$

IC (m^4)	AC (m^2)	IP	P_e (ton)	IB/IC	P_{cr} (ton)	k
I	A	1	612.11	1	576.35	0.94
I	A	1	612.11	1/2	561.78	0.92
I	A	1	612.11	1/3	543.84	0.89
I	A	1	612.11	1/4	528.29	0.86
I	A	1	612.11	1/5	513.84	0.84
I	A	1	612.11	1/6	500.18	0.82
2I	2A	1	1224.22	1	1152.70	0.94
2I	2A	1	1224.22	1/2	1124.92	0.92
2I	2A	1	1224.22	1/3	1089.16	0.89
2I	2A	1	1224.22	1/4	1057.76	0.86
2I	2A	1	1224.22	1/5	1028.56	0.84
2I	2A	1	1224.22	1/6	1001.01	0.82
3I	3A	1	1836.33	1	1729.05	0.94
3I	3A	1	1836.33	1/2	1687.38	0.92
3I	3A	1	1836.33	1/3	1633.74	0.89
3I	3A	1	1836.33	1/4	1586.64	0.86
3I	3A	1	1836.33	1/5	1542.84	0.84
3I	3A	1	1836.33	1/6	1501.52	0.82

Table (5) Effect of changing the arrangement of the lacing bars and decreasing its dimensions on P_{cr} and K for Fixed-Free columns with $IP = 1{:}4$

IC (m^4)	AC (m^2)	IP	P_e (ton)	IB/IC	P_{cr} (ton)	k	Total wt. (ton)
I	A	1	612.11	1	576.35	0.94	0.9668
I	A	1	612.11	1/2	561.78	0.92	0.7052
I	A	1	612.11	1/3	543.84	0.89	0.6180
I	A	1	612.11	1/4	528.29	0.86	0.5744
I	A	1	612.11	1/5	513.84	0.84	0.5484
I	A	1	612.11	1/6	500.18	0.82	0.5308
I	A	2	612.11	1	736.47	1.20	1.7856
I	A	2	612.11	1/2	657.63	1.07	1.0780
I	A	2	612.11	1/3	625.04	1.02	0.8420
I	A	2	612.11	1/4	604.26	0.99	0.7240
I	A	2	612.11	1/5	588.38	0.96	0.6532
I	A	2	612.11	1/6	575.11	0.94	0.6060
I	A	3	612.11	1	594.02	0.97	1.2628
I	A	3	612.11	1/2	572.90	0.94	0.8164
I	A	3	612.11	1/3	554.44	0.91	0.6676
I	A	3	612.11	1/4	537.43	0.88	0.5932
I	A	3	612.11	1/5	521.53	0.85	0.5484
I	A	3	612.11	1/6	506.61	0.83	0.5188
I	A	4	612.11	1	685.93	1.12	1.5668
I	A	4	612.11	1/2	642.57	1.05	0.9684
I	A	4	612.11	1/3	613.57	1.00	0.7688
I	A	4	612.11	1/4	589.46	0.96	0.6692
I	A	4	612.11	1/5	568.12	0.93	0.6092
I	A	4	612.11	1/6	548.75	0.90	0.5692

Table (6) The effect of gusset plate on P_{cr} and K for Pinned-Ended columns having $IP = 1:4$ and $IB/IC = 1/6$

IC m^4	AC m^2	IP	P_e ton	Per	TG mm	P_{cr} ton	k	% P_{cr}
I	A	1	2448.44	0.05	8	1565.69	0.64	3.88
I	A	1	2448.44	0.1	8	1722.15	0.70	14.26
I	A	1	2448.44	0.15	8	1944.19	0.79	28.99
I	A	1	2448.44	0.05	10	1573.78	0.64	4.42
I	A	1	2448.44	0.1	10	1739.26	0.71	15.40
I	A	1	2448.44	0.15	10	1969.31	0.80	30.66
I	A	1	2448.44	0.05	12	1580.39	0.65	4.86
I	A	1	2448.44	0.1	12	1752.46	0.72	16.27
I	A	1	2448.44	0.15	12	1989.19	0.81	31.98
I	A	2	2448.44	0.05	8	1985.54	0.81	5.93
I	A	2	2448.44	0.1	8	2156.23	0.88	15.03
I	A	2	2448.44	0.15	8	2407.04	0.98	28.41
I	A	2	2448.44	0.05	10	1997.18	0.82	6.55
I	A	2	2448.44	0.1	10	2179.03	0.89	16.25
I	A	2	2448.44	0.15	10	2443.66	1.00	30.37
I	A	2	2448.44	0.05	12	2006.77	0.82	7.06
I	A	2	2448.44	0.1	12	2198.00	0.90	17.26
I	A	2	2448.44	0.15	12	2471.82	1.01	31.87
I	A	3	2448.44	0.05	8	1510.75	0.62	4.50
I	A	3	2448.44	0.1	8	1660.84	0.68	14.88
I	A	3	2448.44	0.15	8	1866.01	0.76	29.08
I	A	3	2448.44	0.05	10	1518.58	0.62	5.04
I	A	3	2448.44	0.1	10	1676.89	0.68	15.99
I	A	3	2448.44	0.15	10	1899.13	0.78	31.37
I	A	3	2448.44	0.05	12	1524.90	0.62	5.48
I	A	3	2448.44	0.1	12	1689.30	0.69	16.85
I	A	3	2448.44	0.15	12	1917.87	0.78	32.66
I	A	4	2448.44	0.05	8	1501.02	0.61	4.67
I	A	4	2448.44	0.1	8	1629.62	0.67	13.64
I	A	4	2448.44	0.15	8	1807.76	0.74	26.06
I	A	4	2448.44	0.05	10	1511.50	0.62	5.40
I	A	4	2448.44	0.1	10	1648.56	0.67	14.96
I	A	4	2448.44	0.15	10	1838.79	0.75	28.22
I	A	4	2448.44	0.05	12	1519.66	0.62	5.97
I	A	4	2448.44	0.1	12	1662.89	0.68	15.96
I	A	4	2448.44	0.15	12	1865.36	0.76	30.07

Table (7) The effect of gusset plate on P_{cr} and K for Fixed-Free columns with $IP = 1:4$ and $IB/IC = 1/6$

IC m^4	AC m^2	IP	P_e ton	Per	TG mm	P_{cr} ton	k	% P_{cr}
I	A	1	612.11	0.05	8	518.95	0.85	3.75
I	A	1	612.11	0.1	8	572.88	0.94	14.53
I	A	1	612.11	0.15	8	641.47	1.05	28.25
I	A	1	612.11	0.05	10	523.25	0.85	4.61
I	A	1	612.11	0.1	10	579.39	0.95	15.84
I	A	1	612.11	0.15	10	651.23	1.06	30.20
I	A	1	612.11	0.05	12	525.92	0.86	5.15
I	A	1	612.11	0.1	12	584.49	0.95	16.86
I	A	1	612.11	0.15	12	658.99	1.08	31.75
I	A	2	612.11	0.05	8	602.03	0.98	4.68
I	A	2	612.11	0.1	8	655.64	1.07	14.00
I	A	2	612.11	0.15	8	729.05	1.19	26.77
I	A	2	612.11	0.05	10	605.38	0.99	5.26
I	A	2	612.11	0.1	10	663.12	1.08	15.30
I	A	2	612.11	0.15	10	741.23	1.21	28.89
I	A	2	612.11	0.05	12	608.15	0.99	5.75
I	A	2	612.11	0.1	12	669.02	1.09	16.33
I	A	2	612.11	0.15	12	750.59	1.23	30.51
I	A	3	612.11	0.05	8	531.37	0.87	4.89
I	A	3	612.11	0.1	8	580.62	0.95	14.61
I	A	3	612.11	0.15	8	648.22	1.06	27.95
I	A	3	612.11	0.05	10	534.32	0.87	5.47
I	A	3	612.11	0.1	10	587.17	0.96	15.90
I	A	3	612.11	0.15	10	658.84	1.08	30.05
I	A	3	612.11	0.05	12	536.75	0.88	5.95
I	A	3	612.11	0.1	12	592.31	0.97	16.92
I	A	3	612.11	0.15	12	666.99	1.09	31.66
I	A	4	612.11	0.05	8	566.93	0.93	3.31
I	A	4	612.11	0.1	8	609.78	1.00	11.12
I	A	4	612.11	0.15	8	669.48	1.09	22.00
I	A	4	612.11	0.05	10	570.12	0.93	3.90
I	A	4	612.11	0.1	10	616.94	1.01	12.43
I	A	4	612.11	0.15	10	681.80	1.11	24.25
I	A	4	612.11	0.05	12	572.76	0.94	4.38
I	A	4	612.11	0.1	12	622.83	1.02	13.50
I	A	4	612.11	0.15	12	691.77	1.13	26.06

Table (8) The result from the Fortran program vs. the Sap2000 program

K/K0 for Fixed-Free LG=.05L IP=3 P=100ton				
n	**Without**	**TG = 8mm**	**TG = 10mm**	**TG = 12mm**
0	1	1	1	1
1	0.831	0.839	0.840	0.842
2	0.660	0.675	0.678	0.680
3	0.488	0.510	0.513	0.516
4	0.314	0.343	0.347	0.351
5	0.137			0.186
n = (for K/K0 = 0)	5.794	6.091	6.133	6.144
P_{cr} (Fortran) ton = n*P	579.448	609.068	613.308	614.440
P_{cr} (Sap2000) ton =	577.432	601.920	605.392	608.308
error % (Fortran – Sap) / Sap=	0.35	1.19	1.31	1.01

Fig. (1) Arrangement of Lacing bars for Pinned-Ended columns

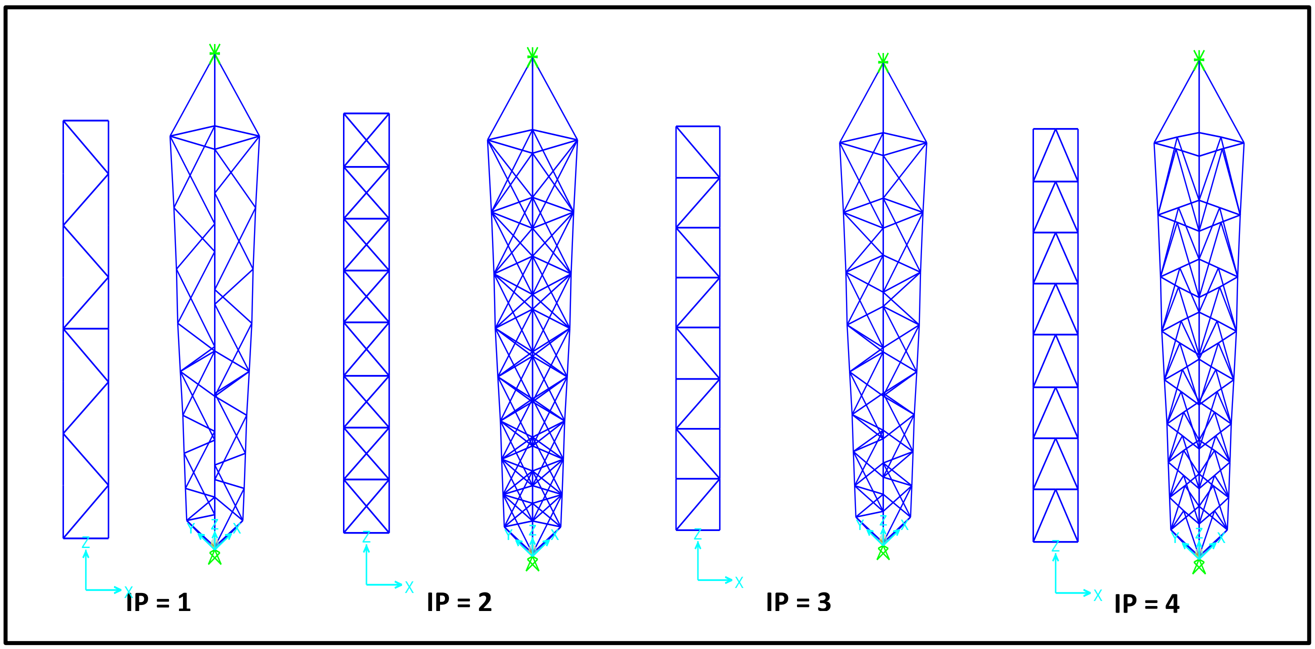


Fig. (2) Arrangement of Lacing bars for Fixed-Free columns

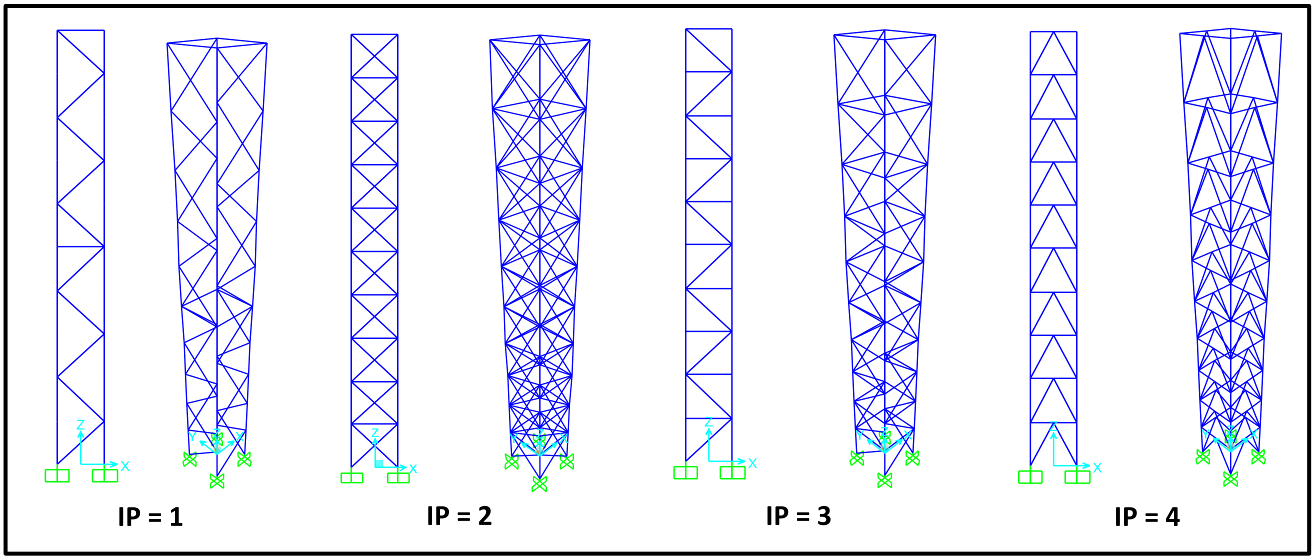


Fig. (3) Variation of Factor K vs. $\frac{IB}{IC}$ for Pinned-Ended columns having $IP = 1:4$

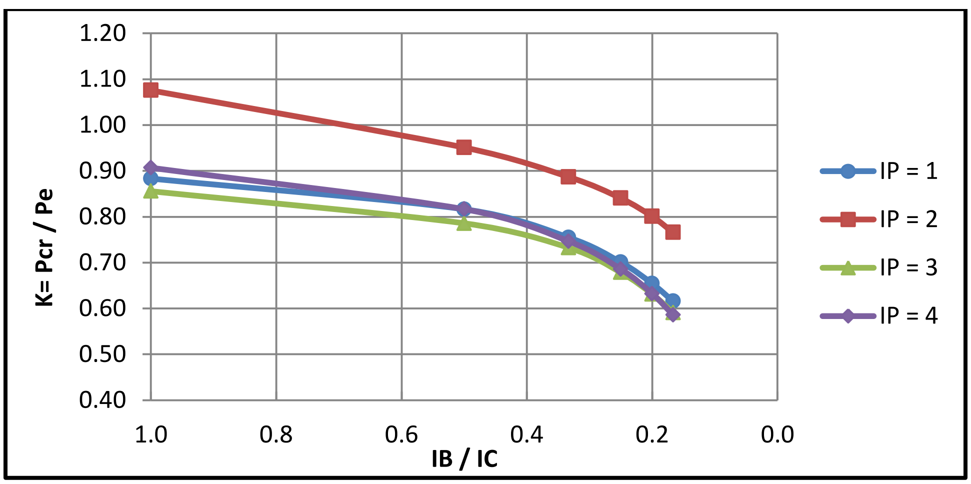


Fig. (4) The relation between the Total weight and IB/IC for Pinned-Ended columns having $IP = 1:4$

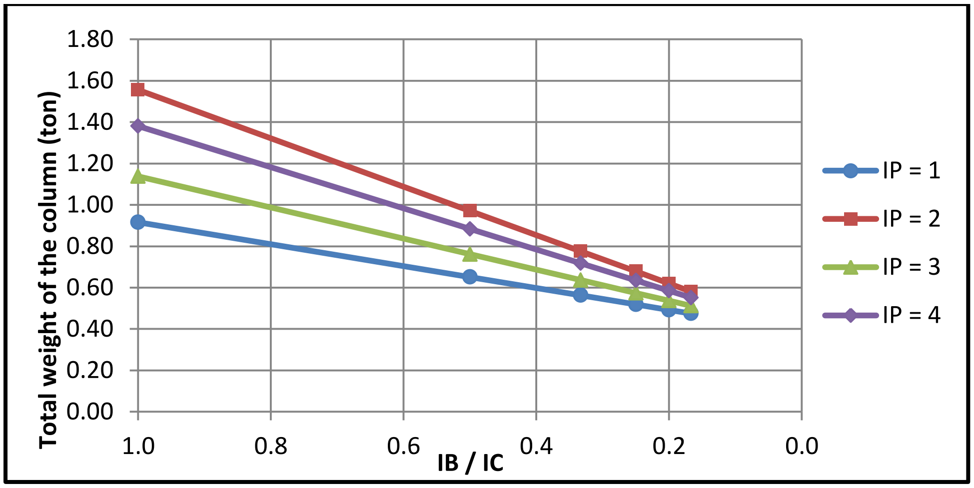


Fig. (5) Variation of critical load P_{cr} vs. IB/IC for Pinned-Ended columns having $IP = 1:4$

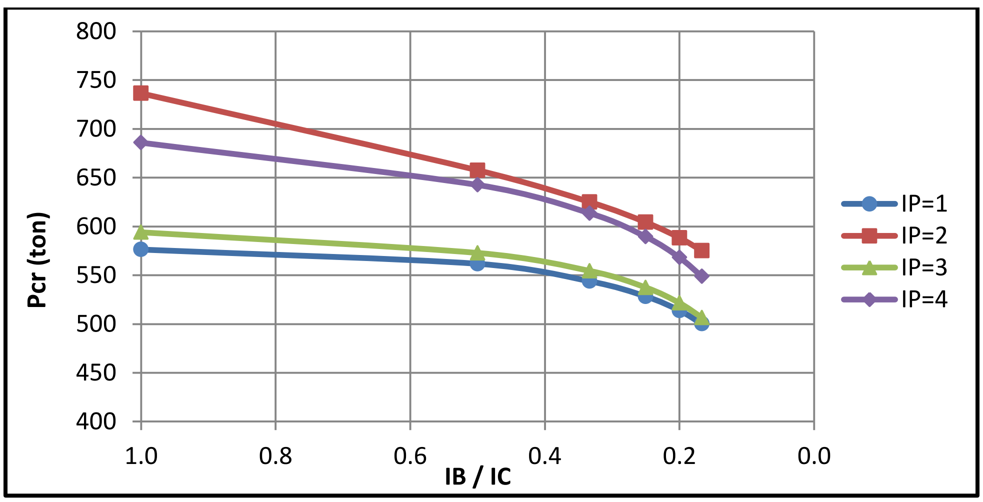


Fig. (6) Buckling shape of Pinned-Ended columns

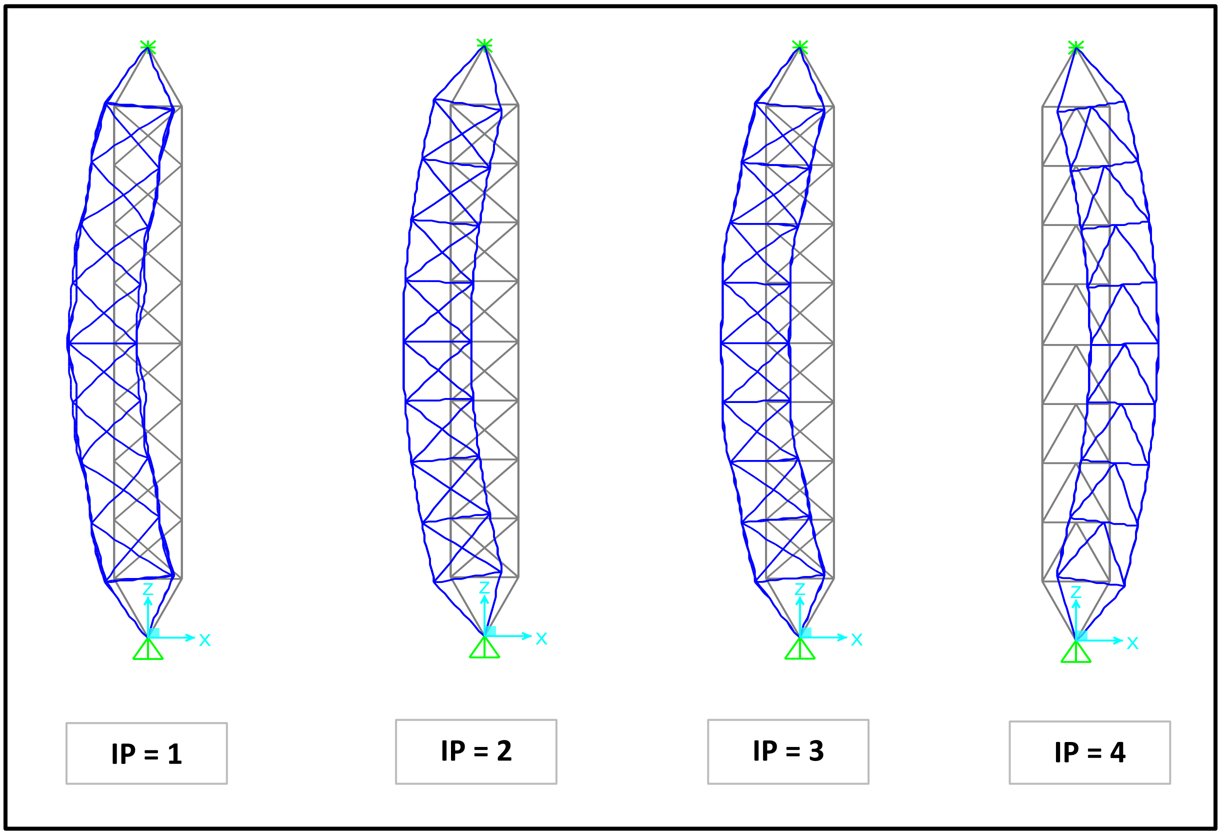


Fig. (7) Variation of Factor K vs. IB/IC for Fixed-Free columns having $IP = 1:4$

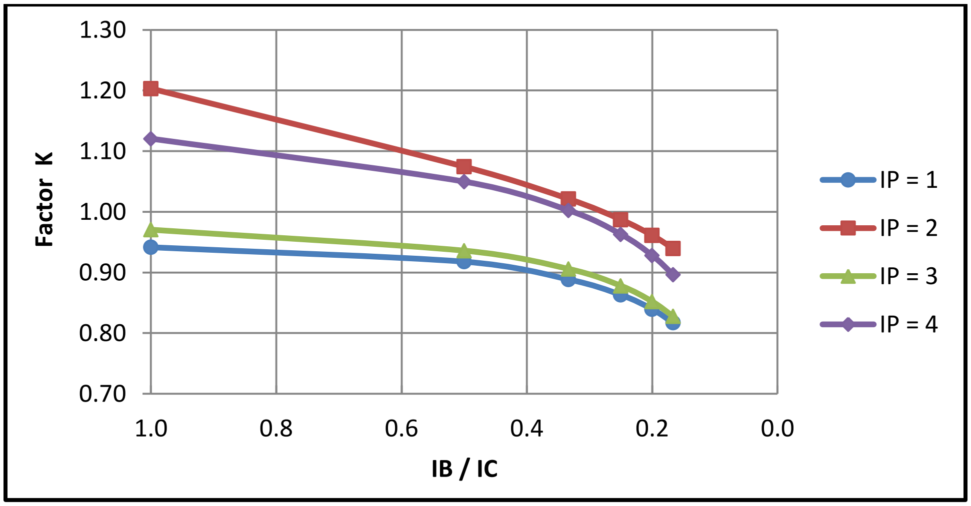


Fig. (8) The relation between the Total weight and IB/IC for Fixed-Free columns having $IP = 1:4$

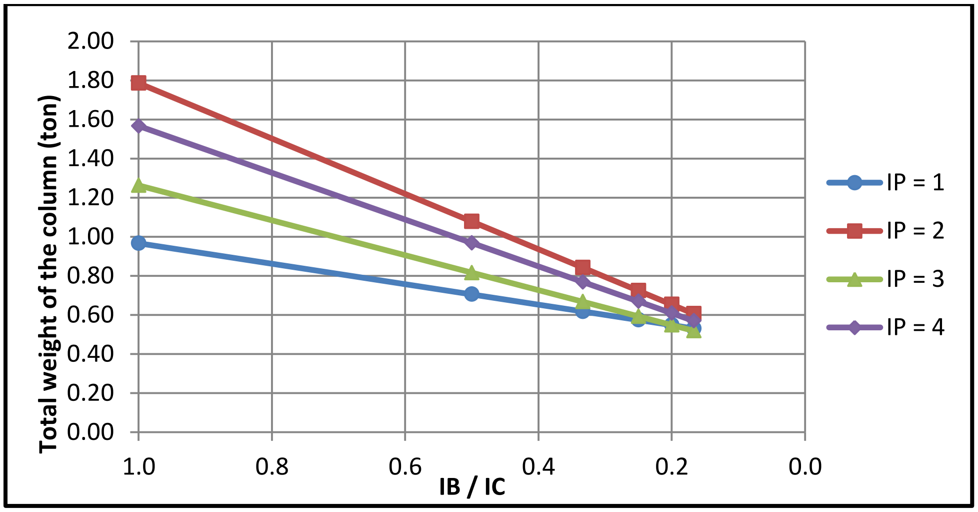


Fig. (9) The variation of critical load P_{cr} vs. $\frac{IB}{IC}$ for Fixed-Free columns having $IP = 1:4$

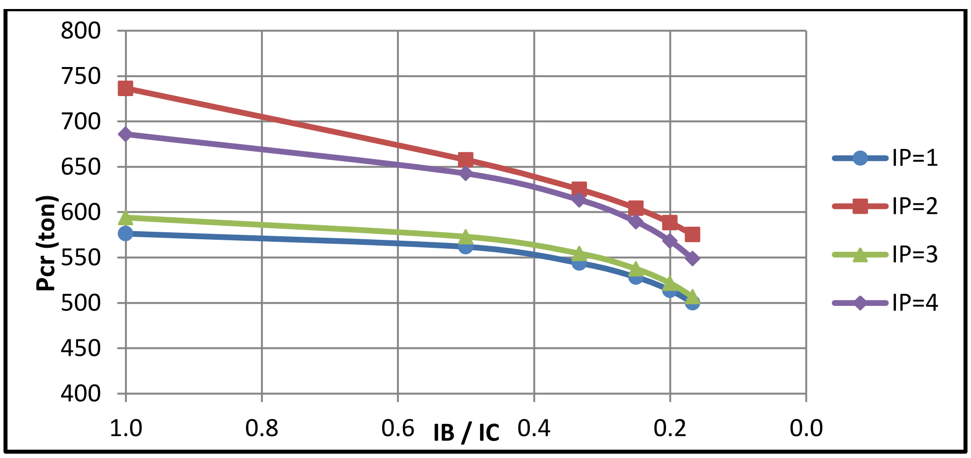


Fig. (10) Buckling shape of Fixed-Free columns

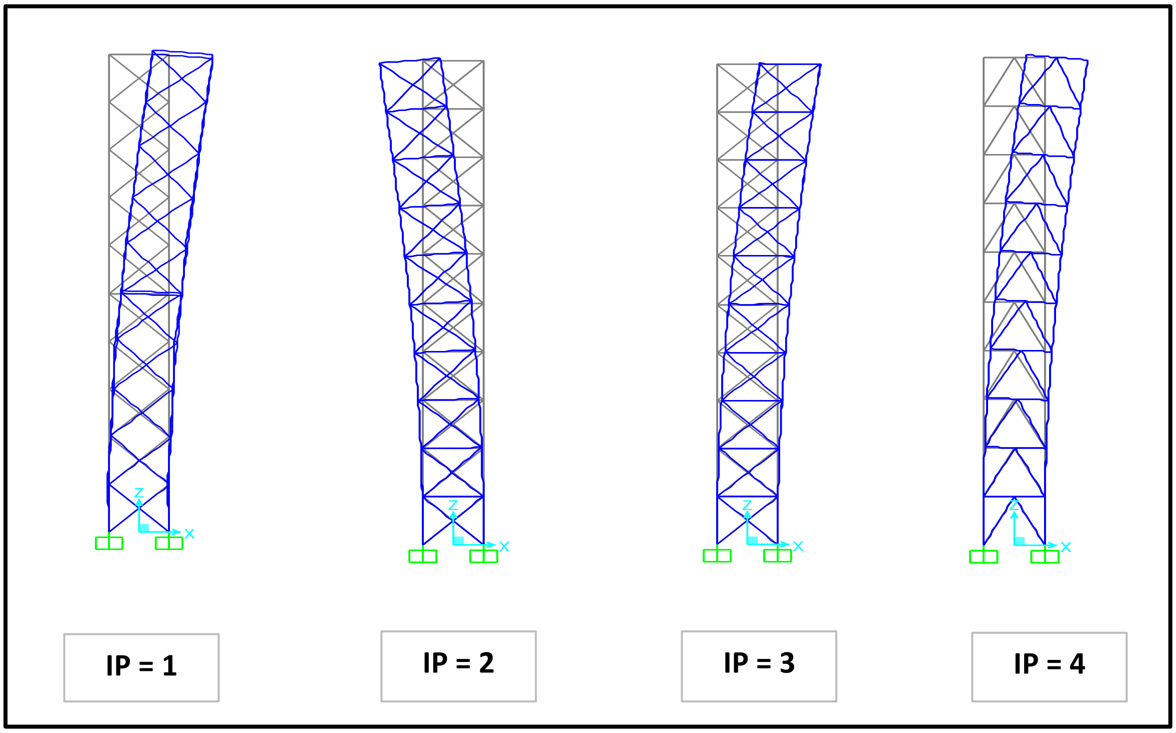
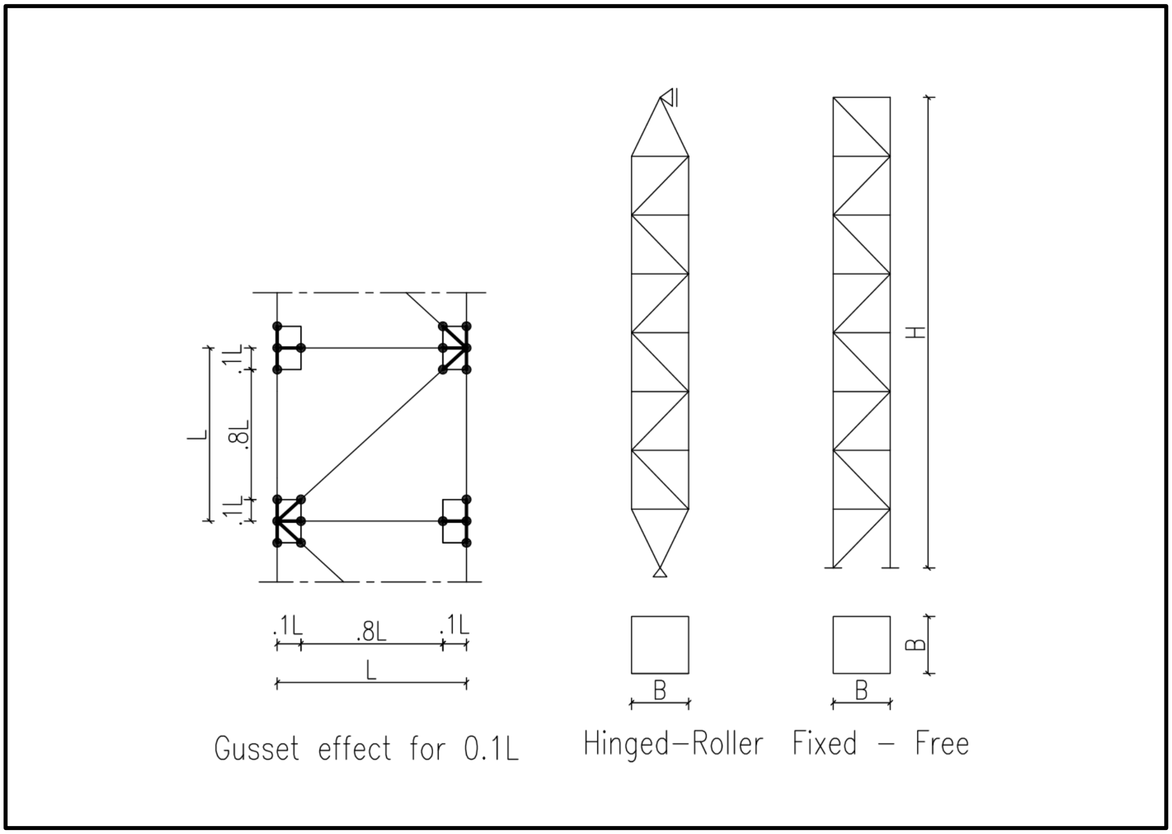


Fig. (11) Gusset plate effect for the case of 0.1L for IP = 3



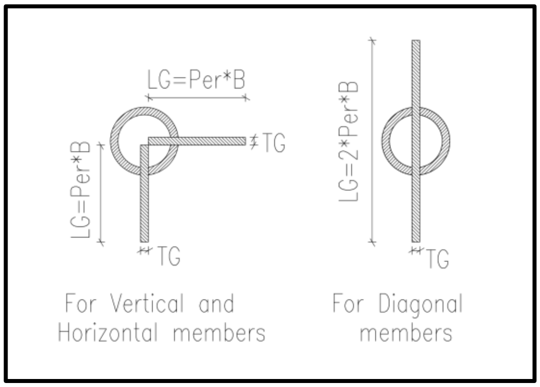


Fig. (12) Cross section at connections of columns having IP = 1, 2 and 3

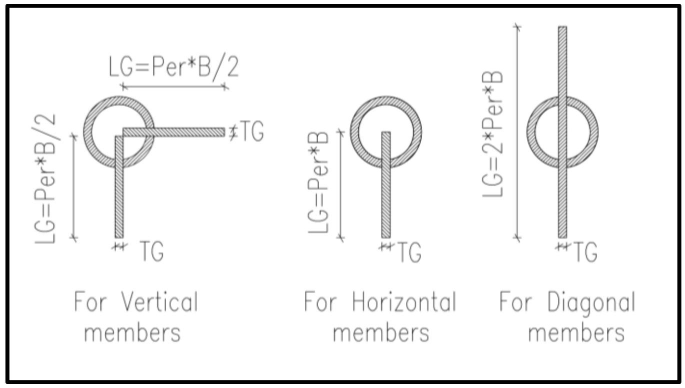


Fig. (13) Cross section at connections of columns having IP = 4

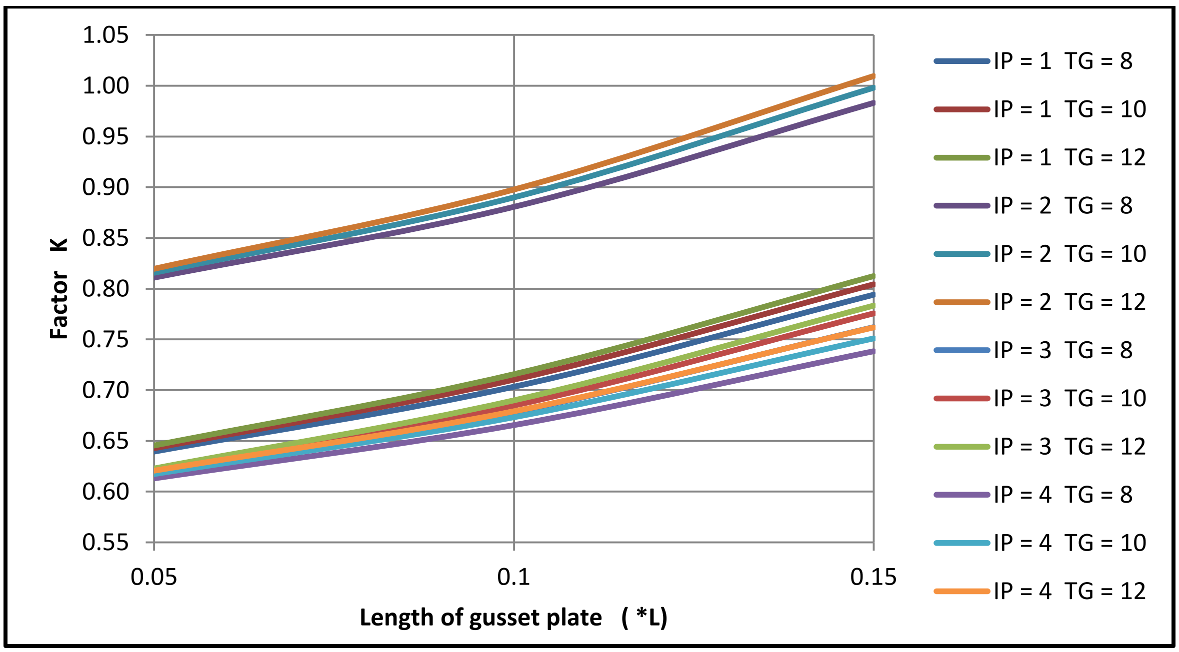


Fig. (14) The effect of gusset plate on K for Pinned-Ended columns having $IB/IC = 1/6$

Fig. (15) The effect of gusset plate on P_{cr} for Pinned-Ended columns having $IB/IC = 1/6$

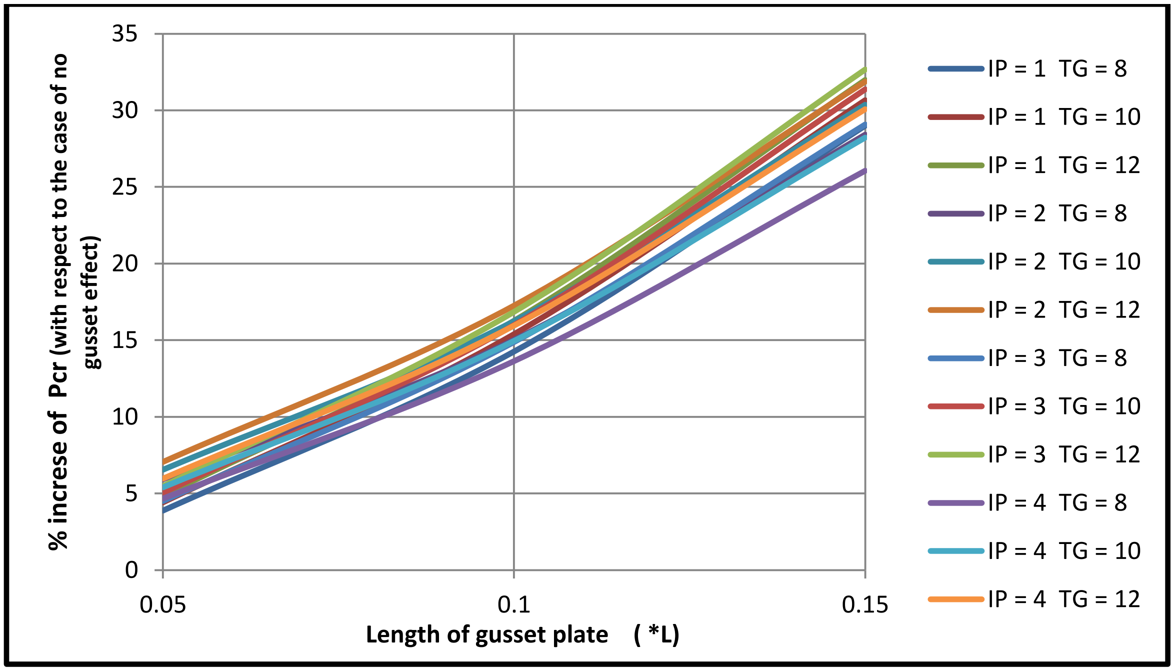


Fig. (16) The effect of gusset plate on K for Fixed-Free columns having $IB/IC = 1/6$

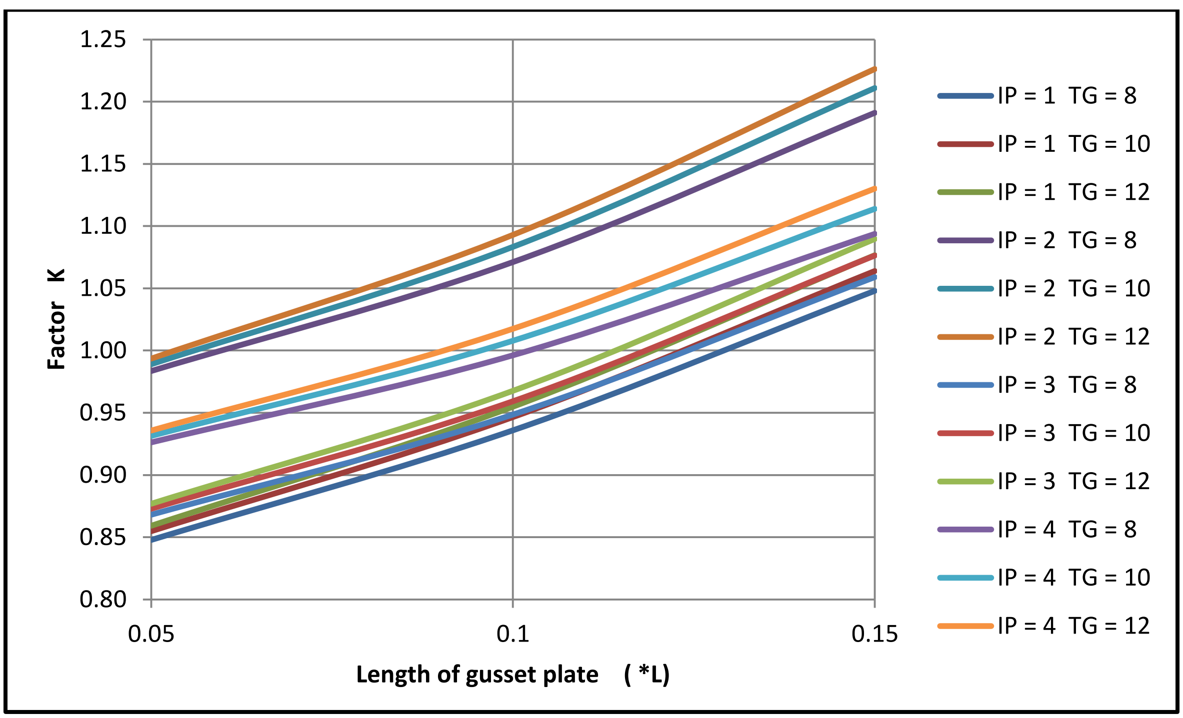


Fig. (17) The effect of gusset plate on P_{cr} for Fixed-Free columns having $IB/IC = 1/6$

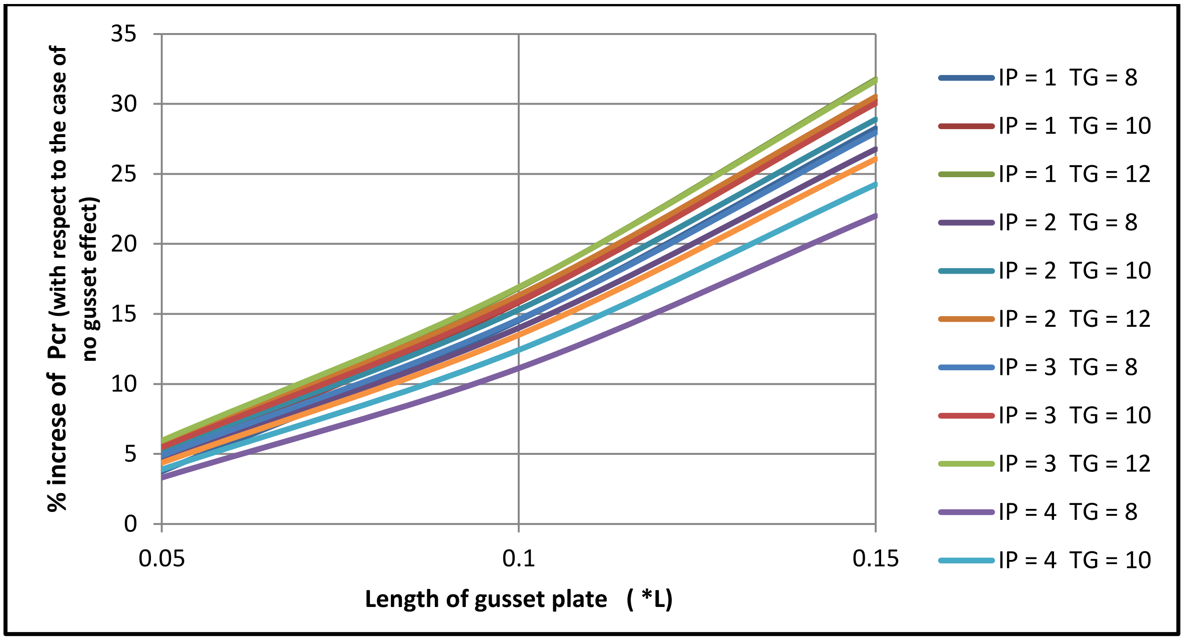


Fig. (18) The result from the FORTRAN program shown in Table (7)