[Mansoura Engineering Journal](https://mej.researchcommons.org/home)

Volume 40 | Issue 1 Article 22

7-6-2020

Studying the Reliability in Multi-Objective Management of Groundwater under Uncertainty of Hydraulic Conductivity Values.

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دراسة درجة الثقة في الإدارة المتعددة الأهداف للمياه الجوفية مع عدم التأكد في قيم التوصيلية الهيدروليكية

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الخلاصة

تعتبر المياه الجوفية مصدر هام للمياه العذبة في كثير من المناطق. فبتزايد السحب الغير منظم من هذا المصدر ينتج عنه مشكلة بيئية خطيرة وهي هبوط مناسيب المياه الجوفية و تداخل دوائر دوران السحب للآبار. يقدم هذا البحث طريقة جديدة للإدارة المثلي للخزانات الجوفية الغير محصورة مع الشك في قيم التوصيلية الهيدروليكية. تحتوي هذه الدراسة على تطبيق نهجين رئيسيين. خلال النهج الأول تم إستخدام طريقة (Monte Carlo Simulation) لتوليد مجموعة متعددة من صور قيم التوصيلية الهيدروليكية المحتملة لمنطقة الدراسة وذلك بناء علي القياسات الحقلية المحدودة. تم تطوير وتطبيق نموذج محاكاة أمثل علي كل صورة من صور قيم التوصيلية الهيدروليكية المستنتجة لمنطقة الدراسة، يعتمد على كل من طريقة العناصر المحدودة لتمثيل سريان المياه الجوفية تحت ظروف سريان مستقر والطريقة الخوارزمية الجينية لتعظيم السحب من الخزان الجوفي وفي نفس الوقت تقليل أعداد الآبار المستخدمة كبديل للتكاليف. ولسهولة التعامل مع النتائج تم تطبيق نموذج رياضي لباحثين آخرين لإستنتاج حل وحيد وسط مناظر لكل صورة محتملة لمنطقة الدراسة و يشمل كل حل على نظام آبار مقترح، به عدد مستنتج لمجموعة آبار و إحداثيات وتصرف كل بئر، و يعد بمثابة حل للمشكلة محل الدراسة مع تحقيق هدفي الدراسة (تعظيم السحب وتقليل أعداد الآبار). و خلال النهج الثاني المقدم في هذه الدراسة تم تطبيق ثلاث طرق (Monte Carlo simulation, Latin Hypercube sampling and First Order Reliability Method) لدراسة درجة الثقة في قيم هدفي الدراسة بعد ربط هذين الهدفين في دالة واحدة نظيركل حل وسطي مستنتج من النهج الأول. تختلف هذه الدراسة عن الدراسات السابقة في التعامل مع المشكلة بإعتبارها مشكلة مزدوجة الأهداف و ليس بتحويلها إلي مشكلة وحيدة الهدف مع دراسة درجة الثقة في الحلول بعد الإنتهاء من إستنتاج جميع الحلول المثلي للمشكلة. تم تطبيق هذه الطريقة علي الخزان الرباعي بوادي الطميلات بمصر. ومن النتائج التي تم الحصول عليها هو إستنتاج نظام آبار وحيد ذو درجة ثقة عالية.

Abstract

Groundwater is considered as an important source of freshwater for several purposes. The increasing demand of groundwater has resulted in an indiscriminate of this source causing environmental hazards such as decline of groundwater levels and well interference. This paper presents a new methodology for optimal management of groundwater in unconfined aquifers in case of uncertainty due to spatial variability of hydraulic conductivities. The suggested methodology includes application of two main consecutive approaches. In the first approach, Monte Carlo simulation is adopted to generate multiple realizations of the hydraulic conductivity values depend on limited field measurements, then a simulation-optimization model is developed and applied to solve the groundwater management problem for each realization. The results of the simulation-optimization model are several Pareto-front optimal solutions for each realization. A unique Pareto-compromise solution for each Pareto-front is determined. In the second approach, to assess the reliability analysis, Pareto-compromise solutions are divided into groups according to the number of wells to detect the most reliable number of wells. The most reliable locations of wells (also their discharges) are detected by splitting these Pareto-compromise solutions into groups according to a new suggested term called radius of gyration. For each group, the performance/state function is assumed as a function of the two objectives corresponding to each Pareto-compromise solution. Then, Monte Carlo simulation, Latin Hypercube sampling, and First Order Reliability Method are applied to study the reliability of the estimated function corresponding to each realization. The methodology is then illustrated by the application on the Quaternary Aquifer of Wadi El-Tumilat, Egypt. The proposed methodology shows its ability to suggest only one system of wells of high level of reliability.

Received: 9 January, 2015 – Revised: 8 February, 2015 - Accepted: 24 February, 2015

Keywords
Reliability, Multi-objective optimization, Uncertainty, Unconfined aquifer, Hydraulic conductivity, Genetic algorithm, Finite element method, Quaternary aquifer.

1. Introduction

Due to the increased irregular extracting of groundwater to meet several life purposes, aquifers depletion may cause serious problems in terms of environment and economic impacts. To extract maximum amount of groundwater, without aquifer depletion, achieving minimum cost, a powerful optimization techniques have to be applied to obtain the best strategy. Optimization techniques are categorized into two types. The first one is deterministic optimization techniques including Linear Programming, Non-Linear Programming, and Dynamic Programming. The second type is the stochastic optimization techniques including Genetic Algorithm (GA), Particle Swarm Optimization, Shuffled Complex Evolution, and Simulating Annealing. These methods were greatly used by several researchers to perform multi-objective management related to groundwater pumping and remediation such as: Park and Aral (2004); Abdel-Gawad (2004 b); Siegfried et al. (2009); Saafan et al. (2011); El-Ghandour and Elbeltagi (2014). GA has been applied extensively to optimize groundwater models. The main advantage of GA is that it uses a population of evolving solutions and identifies several solutions from which the decision maker can select. The main disadvantage lies in the high computational intensity (Djebedjian et al., 2007).

Optimization models are always coupled with simulation models to evaluate the proposed objective functions. In simulation models, numerical approaches [e.g. Finite Element Method (FEM), Finite Difference Method and Boundary Element Method] are applied to simulate groundwater flow and predict the hydraulic heads of the studied aquifer. Within the FEM, any studied aquifer can be considered heterogeneous by adopting different magnitudes of hydraulic conductivity for every element located in the discretized mesh.

Several studies in the literature dealt with groundwater management under parameter uncertainty. Wagner and Gorelick (1987), for example, applied the first and second moment analysis to transfer uncertainty of the hydraulic conductivity to the management problem concerned with groundwater remediation. They applied the chance constrained method to determine best strategy for management under a pre-specified degree of reliability. Sawyer and Lin (1996) repeated the same previous work of Wagner and Gorelick (1987), but with unknown well coordinates (well location). Aly and Peralta (1999) used Artificial Neural Network to simulate the hydraulic response for contaminated aquifers due to different stresses, and then applied GA to find the optimal remediation strategy. Benhachmi et al. (2003) presented a coupled model, for coastal aquifer, consists of simple GA for optimization and chance constrained for reliability. In this model, the location of interface toe was assumed as a function of random variables such as physical parameters and boundary conditions. They concluded that the used methods are practical for making decisions on optimal pumping rates and scenarios exploitation schemes. Baker et al. (2003) studied management of groundwater remediation process under uncertainty in values of hydraulic conductivity. They concluded that increasing the total pumping rate would increase the reliability of the aquifer remediation. Abdel-Gawad (2004 a) used both chance constraint and Monte Carlo methods to study the uncertainty of hydraulic conductivity in coastal aquifers. Multiple realizations of hydraulic conductivity were generated and the optimal design using GA was applied. Ndambuki (2011) studied multi-objective groundwater problem with uncertain parameters as second-order cone optimization problem. He showed that the advantage of this approach was that one does not have to consider a large number of realizations to derive reasonable statistics of the uncertain parameters as the case with Monte Carlo approach. Baú (2012) presented a stochastic optimization framework to assist the planning of groundwater supply systems under uncertain hydraulic conductivity distribution. He structured the framework into a two-objective optimization problem in order to identify the set of pumping designs that trade off the expected management cost against the expected intensity of violation of prescribed hydraulic head constraints.

Parameters uncertainty in groundwater optimization models casts big doubts in the accuracy of the models' output. The Failure in determining the effect of uncertainty in model parameters could considerably reduce the possibility of success of optimization models. The objective of this research is to introduce and apply a new methodology for optimal groundwater management under uncertainty in hydraulic conductivity. In this study, hydraulic conductivity is assumed to be random and log-normally distributed, where the parameters of this distribution are obtained from field data. Multiple realizations of the hydraulic conductivity are generated based on Monte Carlo simulation and the optimization model is then applied for each realization. Furthermore, the reliability analysis is performed for the results of the optimization model. The proposed methodology applies multi-objective GA as an optimization tool; the FEM as hydraulic simulation solver; and Monte Carlo simulation (MCs), Latin Hypercube sampling (LHs) and First Order Reliability Method (FORM) to conduct the reliability analysis. The novelty of the current research stems from dealing with two-objective reliability based optimization of groundwater pumping in unconfined aquifers. The application is carried out at Quaternary aquifer of Wadi El-Tumilat (QAWT), Egypt.

2. The simulation-optimization model

In this study, a FEM simulation model is coupled with a GA optimization model to solve groundwater management problems. The developed coupled model is used to simultaneously establish the maximum discharge for a set of wells from a given aquifer with minimum cost, considering the number of wells, their discharges and their locations as decision variables.

2.1 FEM Flow Simulation Model

In this model, the governing equation describing the three dimensional movement of ground water described as follows (Bear, 1979):

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right)-W+ \\ +\sum_{i=1}^{N_W} Q_i\delta(x-x_i)\delta(y-y_i)=S_s\frac{\partial h}{\partial t} \tag{1}$$

in which, K_{xx}, K_{yy}, K_{zz}: the principal components of hydraulic conductivity aligned along the x, y, and z coordinate axes respectively; h: the hydraulic head; W: the uniform rainfall or uniform evaporation; Q_i: the injection or pumping rate of the i^{th} well; $\delta(z)$: the Dirac delta function which equals 1 if z equal zero otherwise equals zero; N_W: number of field wells; S_s: the specific storage and t: the time.

The following assumptions are taken into consideration: (1) Dupuit's hydraulic assumption is employed to vertically integrate the flow equation, reducing it from three dimensional geometry to two dimensional, (2) aquifer specific storage is ignored such that the governing equation becomes time independent, (3) wells fully penetrate the aquifer thickness, (4) impervious bed of the aquifer is considered horizontal, (5) the vertical and horizontal hydraulic conductivity components are the same (i.e. isotropic aquifer), and (6) unconfined aquifer is considered through the solution.

According to the previous assumptions, Eq. (1) can be re-written as follows:

$$\frac{\partial}{\partial x}\left(Kh\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(Kh\frac{\partial h}{\partial y}\right)= \\ =W-\sum_{i=1}^{N_W}Q_i\delta(x-x_i)\delta(y-y_i) \tag{2}$$

Eq. (2) is a nonlinear form of flow equation that utilizing the hydraulic head of the groundwater as a dependent variable. This equation can be linearized by the substitution ($\phi = h^2/2$) where ϕ is the potential. Accordingly, Eq. (2) can take the following linear form:

$$K\frac{\partial^2\phi}{\partial x^2}+K\frac{\partial^2\phi}{\partial y^2}=W-\sum_{i=1}^{N_W}Q_i\delta(x-x_i)\delta(y-y_i) \tag{3}$$

FEM is adopted in this study to discretize the studied aquifer to number of linear rectangular elements and the governing partial deferential Eq. (3) is solved over each element. Within the FEM, any studied aquifer can be considered heterogeneous by adopting different magnitudes of hydraulic conductivity for each element located in the discretized mesh. Based on the FEM procedures, Eq. (3) can be written in the following matrix form:

$$[A]\times[\varphi]=[F] \tag{4}$$

in which, A: the conductance/stiffness matrix, φ: the unknown vector of potentials and F: the load vector which contains the external source or sink of water and flux concentration.

In this research, linearity of Eq. (4) is exploited by inversing the conductance/stiffness matrix, using the LU-decomposition method, once and using its inversion numerous times through the optimization process by multiplication the inversion and different load vectors of pumping rates from the suggested well system (Abdel-Gawad, 2004 a). It must be noticed that there is a conductance matrix corresponding to each realization of the hydraulic conductivity random field. This procedure significantly decreases the computational time and facilitates studying the reliability.

2.2 GA Optimization Model

GA is a stochastic optimization technique, which was developed by Holland in 1975 (Goldberg, 1989). GA simulates mechanisms of population genetics and natural rules of survival in pursuit of the ideas of adaptation. GA, in the last few years, has shown to be valuable tool for solving complex optimization problems in the field of water resources. The GA based solution method can generate both convex and non-convex points of the trade-off surface, and accommodate non-linearities within the multiple objective functions. GA consists of three basic operations: selection; crossover and mutation. In the proposed GA optimization model, several chromosomes which represent different sets are formed randomly. Every generated chromosome consists of number of codes equal to pre-specified number of well fields. Each code consists of number of genes equal to the number of decision variables (i.e. coordinates and pumping rates). Number of well fields in each chromosome is variable. This is carried out by generating random number between *zero* and *one* for each code. If this number less than 0.5 the well field is turn off otherwise is turn on.

To compute the fitness of each chromosome, a layer classification technique is used whereby the population is incrementally sorting using Pareto dominance. This method can be explained as follows (Ngatchou et al., 2005; and Liu and Hammad, 1997):

- All chromosomes in the current population are compared, according to their objective functions to determine the Pareto optimal set of this population and are assigned a rank of one for this set. A chromosome belongs to this Pareto set if there is no other chromosome that can improve at least one of the objectives without degradation of any other objective.

- The set of chromosomes having rank one is set apart, and the remaining chromosomes are compared to select a new non-dominated/Pareto set with a rank of two.
- This process continues until the entire population is ranked.
- The fitness function value of each chromosome is assigned according to its rank, using the following equation (Liu and Hammad, 1997):

$$F_i = 1/rank_i \qquad (5)$$

in which, F_i: the fitness and $rank_i$: the rank number of individual i.

For the present analyses, the presented model is developed to optimize the two conflicting objectives (i.e. maximizing the pumping rates and minimizing the cost). A real coding of decision variables could be applied in this model. The GA optimization model includes the FEM Flow Simulation Model to carry out the hydraulic analysis using the data included in every chromosome.

3. Model verification

In this section, a typical sample problem was previously solved by El-Ghandour and Elbeltagi (2014), are chosen to verify the proposed simulation-optimization model. The hypothetical aquifer have dimensions of 4500×10000 m^2. This hypothetical aquifer consists of no-flow boundaries on two sides and constant head boundaries on the other two sides. The aquifer is composed of sand and gravel and it is assumed that porous medium is homogeneous and isotropic. The hydraulic conductivity is 50 m/day, the areal recharge is 0.001 m/day, and the constant head equal to 20 m on the two boundaries. Two objective functions are simultaneously optimized. The first one is to maximize the total pumping rates from 10 pumping wells of known locations, while the second objective is to minimize the total pumping cost, which consists of the well drilling, capital, and operating costs. The decision variables of the management problem are the associated pumping rates from the pre-specified system of the ten wells. The constraints set on the management problem: (i) pumping rate from each well subject to specified lower and upper bounds, (ii) hydraulic heads at well locations must be greater than a specified lower bound, (iii) total pumping from the aquifer must exceed the given demand.

The sensitivity analysis is carried out to determine the suitable values of GA parameters. These values are found as follows: population size = 100; maximum number of generations = 300; crossover ratio = 0.8; mutation ratio = 0.05 and a uniform crossover is adopted. After applying the model to this hypothetical problem, it is found that the FEM-GA solution converged to the optimal or near optimal solutions (i.e. Pareto front) after 120 generations. The Pareto fronts generated by the present model and the corresponding one given by El-Ghandour and Elbeltagi (2014) are compared as shown in Figure (1). It can be seen, from this figure, that the obtained Pareto front is nearly coincide with that presented by El-Ghandour and Elbeltagi (2014). The small deviation shown between the two Pareto fronts may be due to the difference of hydraulic solvers used in the two models. The hydraulic solver in the proposed model is dependent on the numerical solution of groundwater equation using FEM, while the corresponding one given by El-Ghandour and Elbeltagi (2014) is dependent on the analytical solution of groundwater equation, Eq. (3).

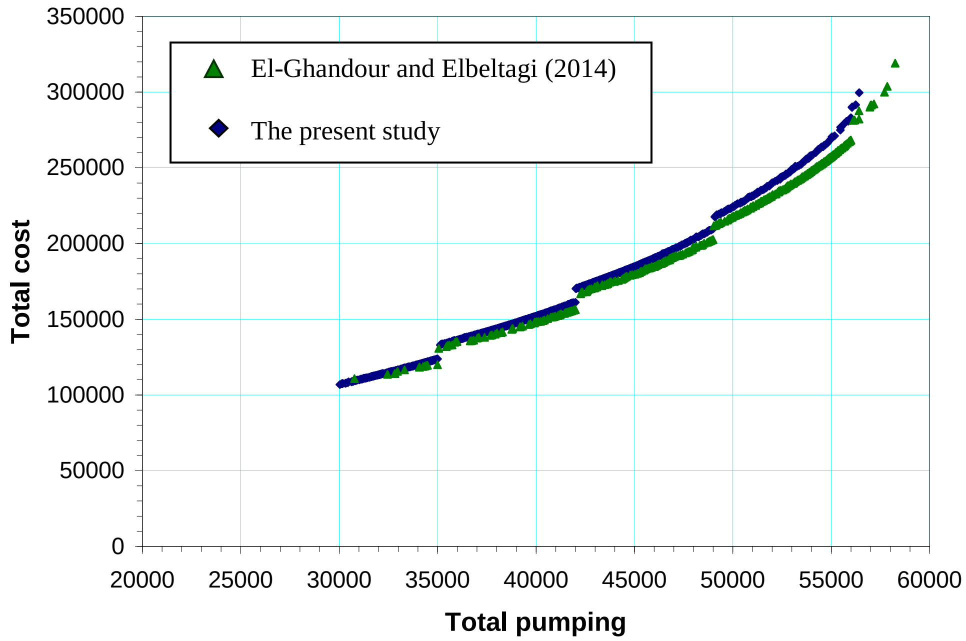

Figure 1: Comparison between Pareto fronts generated by the present model and the corresponding optimal one given by El-Ghandour and Elbeltagi (2014)

4. Reliability analysis

Groundwater management is generally carried out in an environment of uncertainties like any other resources management. Heterogeneity in natural aquifers formations is widely recognized as one of the major factors contributing to uncertainty in predicting groundwater flow behavior and management strategies. The following three methods are adopted here to study the reliability in multi-objective management of groundwater.

Monte Carlo sampling (MCs): MCs given by Madsen et al. (1986) consists of drawing samples of the basic variables according to their probabilistic characteristics and then feeding them into a function called performance/state function (g). An estimate of the probability of failure P_f can be found as follows:

$$P_f = \frac{N_f}{N} \tag{6}$$

in which, N_f: number of simulation cycles in which ($g < 0$) and N: total number of simulation cycles.

As N approaches infinity, P_f approaches the true probability of failure. It is recommended to measure the statistical accuracy of the estimated probability of failure by computing its Coefficient of Variation (COV) as follows (Ayyub and McCuen, 2002):

$$COV = \frac{\sqrt{(1-P_f)P_f/N}}{P_f} \tag{7}$$

The smaller the COV, the better the accuracy of the estimated probability of failure. As mentioned above, it is evident from Eq. (6) that as N approaches infinity, $COV(P_f)$ approaches to zero.

Latin Hypercube sampling (LHs): LHs is designed to accurately recreate the input distribution through sampling in less iteration when compared with MCs. It depends on a technique known as “stratified sampling without replacement” which initially given by Iman et al. (1980) and having the following steps:

- The probability distribution is divided into n intervals of equal probability, where n is the number of iterations that are to be performed on the model. In the first iteration, one of these intervals is selected using a random number.

- A second random number is then generated to determine where, within that interval, the cumulative distribution function $F(x)$ should lay.
- The input variable x is equal to the inverse function (i.e. $G\,[F(x)]$).
- The process is repeated for the second iteration but the interval used in the first iteration is marked as having already been used and therefore will not be selected again.
- This process is repeated for all of the iterations.

First Order Reliability Method (FORM): FORM, given by Melchers (1999), consists of the following steps, Figure (2):

- Transformation of the space of the basic random variables X_1, X_2,…, X_n into a space of standard normal variables U_1, U_2,…, U_n.

where,

$$U_i = \frac{x_i - \mu_{X_i}}{\sigma_{X_i}} \tag{8}$$

- Determination of the state function surface limits by putting g (U) = 0.
- In this transformed space, search the point of minimum distance from the origin on the limit state surface u^* (the design point).
- Approximation of the failure surface near the design point.
- Computation of the failure probability corresponding to the approximating failure surface.

The probability of failure is estimated as follows:

$$P_f = \Phi(-\beta_{HL}) \tag{9}$$

where, Φ is related to cumulative distribution of the standard normal law and β_{HL} is the Hasofer-Lind reliability index (Hasofer and Lind, 1974). The precision of this approximation depends on the non-linearity of the failure surface.

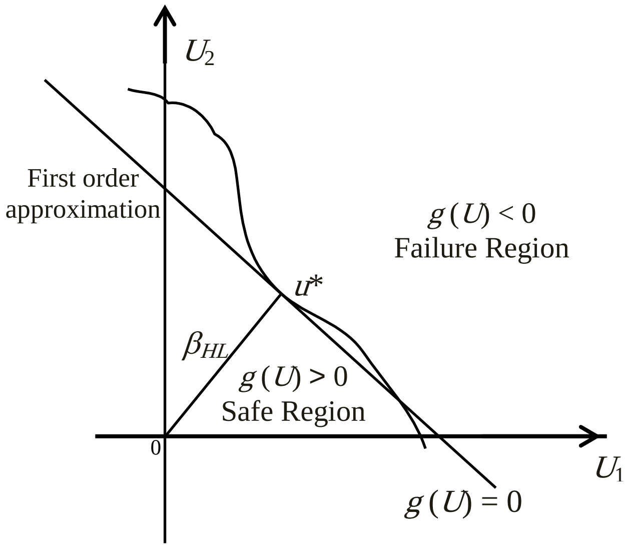

Figure 2: Reliability assessment with FORM method (adapted from Lee, 2008)

5. Compromise solution

Results of any multi-objective problem are the series of Pareto-optimal solutions called Pareto-front. Each Pareto-optimal solution in this Pareto-front is considered a solution for the problem under study depending on the decision maker opinion. A unique solution has to be considered, from Pareto-optimal solutions, to possible study its reliability. In this study, a Pareto-compromise solution is determined to express the required unique solution. To obtain the unique Pareto-compromise solution of multi-objective optimization, a technique based on a theorem proposed by Grierson (2008) is used. This is from a set of Pareto optimal solutions for which the competing criteria/objectives are mutually satisfied in a Pareto optimal sense. This technique is called Multi-Criteria Decision Making (MCDM) strategy. The theorem is called Pareto–Edgeworth–Grierson (PEG). The mathematical formulations used to determine the compromise solution among a set of Pareto – optimal solutions, are programmed in a code given by Elbeltagi et al. (2010).

6. Real field application

After verifying the simulation-optimization model against the hypothetical problem, it is applied to the Quaternary Aquifer of Wadi El-Tumilat (QAWT), Egypt. The developed model is applied to simultaneously maximizing the total pumping rates and minimizing the number of wells, a surrogate of initial cost, by identifying the optimal location and discharge of wells in addition to the number of wells.

6.1 Site Description

The Quaternary Aquifer of Wadi El-Tumilat (QAWT) lies between latitudes $30^{o}25^{\backslash}$ and $30^{o}35^{\backslash}$ N and longitude $31^{o}45^{\backslash}$ and $32^{o}20^{\backslash}$ E. It is bounded on the North-west by Ismailia canal, on the west by Wadi El-Watan, on the east by Suez Fresh water Canal and on the south by Cairo-Shubrawit Ridges with a total area of 800 km^2, Figure (3). The QAWT is characterized by desert climate, with arid, hot and rainless summer, and mild winter with low precipitation (22-40 mm/year). The evaporation rate is very high (6-12 mm/day). The water bearing formation in the Wadi El-Tumilat area comprise the QAWT, occupies the shallow zone and the Miocene aquifer dominating the deeper part. The QAWT represents the main aquifer in the region and composed of fluviatile and fluviomarine graded sand and gravel with clay intercalations of limited extension. The basal portion of this aquifer is formed of dark plastic clay. The Quaternary deposits rest directly with unconformity surface on the Miocene hard limestone as recognized in the north and south of wadi El-Tumilat. Its total thickness increases generally from south to north. The Miocene aquifer is dominated by clastic facies in the southern part of the study area and overlain by about 200 m of Quaternary deposits. In Belbies – El-Tell El-Kabier – El-Salhiya fluviatile plain, the Miocene sediments are composed of alternating sandy limestone and clay lenses, loose quartz sand and marl. The aquifer is more clayey towards east. In the narrow strip adjacent to the Ismailia canal, the depth to the groundwater is highly affected by the surface water running in the canal.

The groundwater flow in the QAWT is directed mainly from south to north in the southern part (Miocene aquifer) with very low hydraulic gradient ($\approx 2\times10^{-4}$). An opposite direction is recorded from north to south in the area lying south of Ismailia canal (hydraulic gradient $\approx$ 4x 10^{-4}). Along the main flood plain and downstream of Wadi El-Tumilat, an opposite direction is recorded from north to south (local flow) in the area lying south of Ismailia canal (the hydraulic gradient is about 8×10^{-4}). The main groundwater recharging source is the Ismailia canal while Suez and El-Manaief fresh water canals are additional sources.

Complete surveying of 28 selected groundwater points were performed in the field during the year 2006. In order to study the multi-objective groundwater management, four pumping tests and five infiltration tests were carried out to estimate the values of QAWT hydraulic conductivity. These values are found to be 1.78, 6.83, 1.15, 6.03, 1.0, 9.07, and 4.38 m/day.

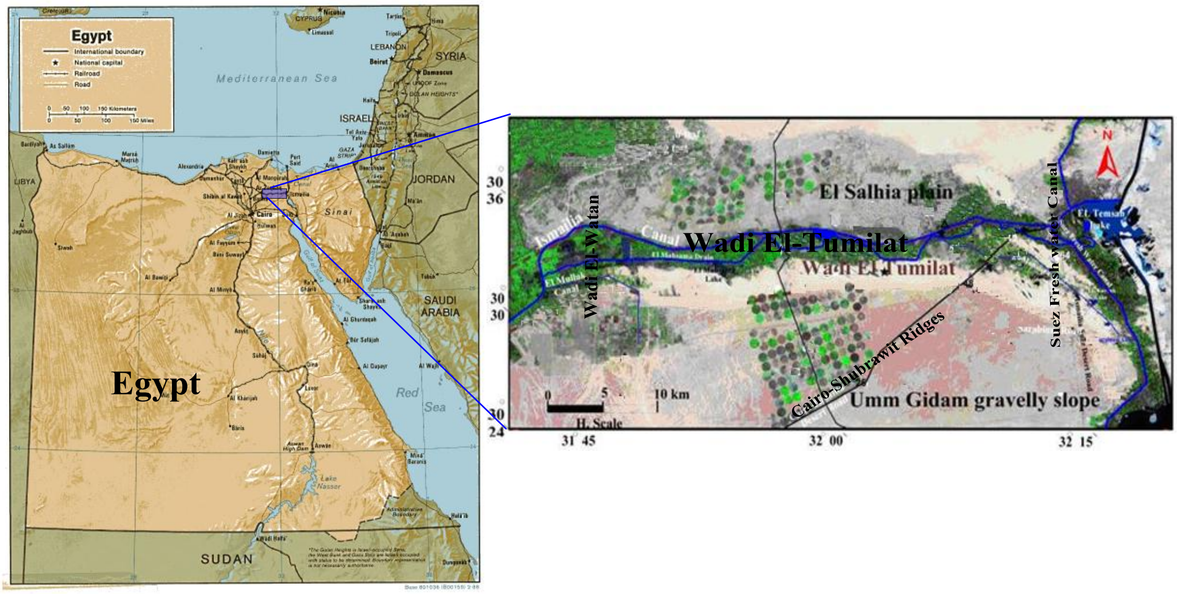

Figure 3: Location map of the Quaternary Aquifer of Wadi El Tumilat (QAWT)

6.2 Model Formulations for the QAWT

The model formulations for the study area are given as follows:

$$f_1 = \max \left[\sum_{i=1}^{N_W} Q_i - \lambda P(h) \right] \quad (10)$$

$$f_2 = \min \left[N_W + \alpha P(h) \right] \quad (11)$$

Subjected to:

$$h_i \geq h_{imin}, \qquad i = 1, 2, 3, \ldots N_W \quad (12)$$

$$Q_{i\,min} \leq Q_i \leq Q_{i\,max}, \; i = 1, 2, 3, \ldots N_W \quad (13)$$

$$P(h) = \begin{cases} h_{i\min} - h_i & if \quad h_i < h_{i\min} \\ 0 & if \quad h_i \geq h_{i\min} \end{cases}$$
$$i = 1, 2, 3, \ldots N_W \quad (14)$$

in which, Q_i: the pumping rate of well i; N_W: the number of field wells; $P(h)$: the penalty terms associated with permissible hydraulic heads at well locations; $h_{i\,min}$: the minimum hydraulic head value at well i; $Q_{i\,min}$ and $Q_{i\,max}$: the minimum and maximum bounds of the pumping rates at well i; and λ and α: weighting factors.

6.3 Steps of Solution

The steps of applying the current methodology are described as follows:

- Generate 1000 realization of the hydraulic conductivity field dependent on field measurements using Monte Carlo simulation.
- Apply GA model for all realizations to find Pareto optimal solutions set corresponding to each realization
- Apply compromise solution model to determine a unique Pareto compromise solution for each Pareto optimal solutions set deduced from previous step. Each compromise solution contains suggested well system (consists of number of wells; and coordinates and discharge for each well) for each realization.
- The two objective function values in each compromise solution are the minimum number of pumping wells and the total discharge of the wells.
- After completing the previous steps, 1000 compromise solutions are available.
- These 1000 compromise solutions are divided into groups according to the number of wells.
- The reliability analysis is conducted for each group using MCs, LTs and FORM

and the reliable number of wells is detected.

- The obtained compromise solutions (1000 solutions) are divided also into groups according to the new used term called radius of gyration, Eq. (15), explained later.
- MCs, LTs and FORM are used to detect the most reliable radius of gyration.
- Finally, a single reliable optimum solution is obtained to help decision maker.

6.4 Results and Discussion

Figure (4) shows the Pareto-compromise solutions corresponding to all hydraulic conductivity realizations. Each solution shown in this figure is considered a solution for the problem under consideration.

The probability of failure P_f estimated from MCs and LHs; and β_{HL} obtained from FORM are taken as an indication of the reliability of the optimized solutions.

In order to check the reliability of the results of the optimization model, the compromise solutions obtained from the optimization model are divided into six groups as shown in Table (1) according to the number of wells, N_W (six groups G_{Ni}, i=1, 2, 3, 4, 5, 6) and into five groups as shown in Tables (2) according to the radius of gyration, R_g (five groups G_{Ri}, i=1, 2, 3, 4, 5).

The radius of gyration R_g of the points which represent the locations of the wells is taken as an expression of the spacing of wells. R_g is calculated as follows:

$$R_g = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(r_i - r_m)^2} \tag{15}$$

in which, r_i: position of well i from the center of gravity and r_m: the mean position of the wells.

The limit state function g for each group of compromise solutions is assumed as follows:

$$g = x_1 + x_2 - min\left(\frac{f_{1_i}}{f_{1_{max}}} + \frac{f_{2_{min}}}{f_{2_i}}\right) \tag{16}$$

where x_1 and x_2 are generated (or simulated) from $\frac{f_{1_i}}{f_{1_{max}}}$ and $\frac{f_{2_{min}}}{f_{2_i}}$ respectively, f_{1_i} is the total pumping rate in the compromise solution i, i = 1, 2, 3, …, n in which n is the number of compromise solutions, f_{2_i} is the number of wells in the compromise solution i, $f_{1_{max}}$ is the maximum total pumping rate in the compromise solutions, and $f_{2_{min}}$ is the minimum number of wells in the compromise solutions.

Tables (1) and (2) present the results of the reliability analysis models (MCs, LTs and FORM) for different ranges of number of wells, N_W, and different ranges of radius of gyration, R_g, respectively. These results are also plotted in Figures (5) through (8).

As shown in Figures (5) and (6), the results of MCs, LTs and FORM are consistent with each other. Figure (5) shows that, using the aforementioned methods (MCs and LTs), increasing the number of wells N_W increases the reliability of the optimization model (decreases the probability of failure P_f). The same results may be deduced from Figure (6), which depicts the values of the Hasofer-Lind reliability index, β_{HL} (obtained from FORM method) corresponding to different ranges of number of wells, N_W. The effect of the locations of the wells (radius of gyration, R_g) on the reliability of the optimization model is given in Figures (7) and (8). These Figures show that, as R_g increases, the reliability of the results of the optimization model increases to reach its maximum value at R_g = 3800-4000 m, after that it decreases.

To help decision makers to select the ordinates of the design point, the following steps are assumed:

1. f_2 equal to 58 [average value of 56 and 60 which is corresponding to group G_{N3}, Table (1)] is assumed as the second ordinate of the design point.
2. Search the optimal solutions corresponding to f_2 = 58 and select the one with R_g = 3900 [average value of 3800 and 4000 which is corresponding to group G_{R4}, Table (2)]. The selected solution (f_1 =1260916) is assumed as the design point.

The optimal scheme corresponding to the design point is depicted in Figure (9). Table (3) lists coordinates and pumping rates for the well system shown in Figure (9).

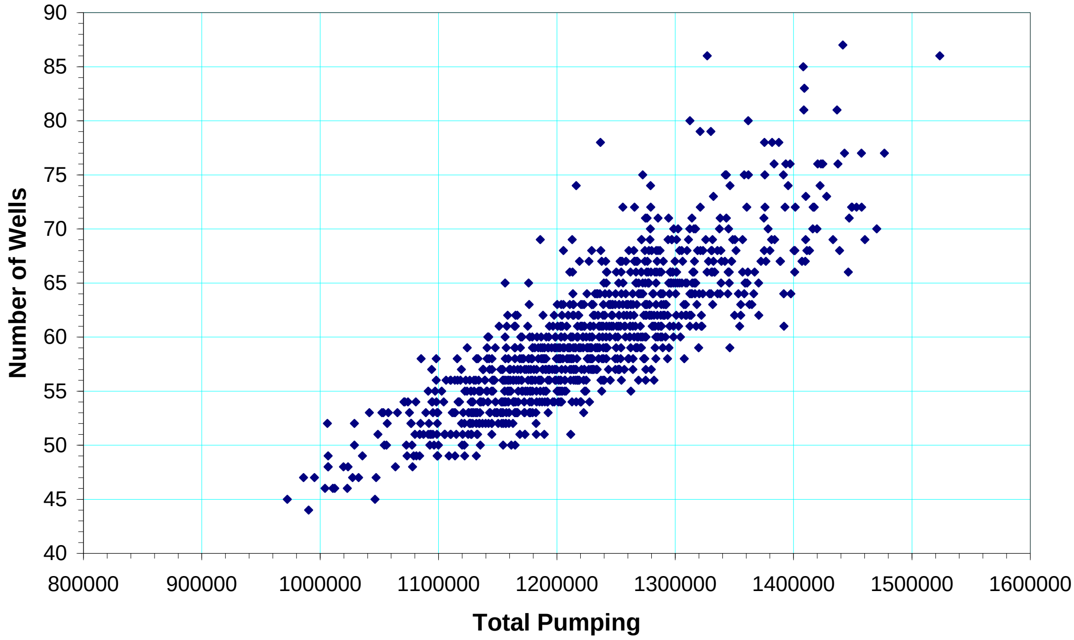

Figure 4: Pareto-compromise solutions corresponding to all hydraulic conductivity realizations

Table 1: Results of the reliability analysis for different number of wells, N_W

		G_{N1}	G_{N2}	G_{N3}	G_{N4}	G_{N5}	G_{N6}
	N_W	**44-50**	**51-55**	**56-60**	**61-65**	**66-70**	**>70**
P_f	**MCs**	0.245	0.027	0.006	0.0013	0.002	0.012
	LHs	0.246	0.019	0.009	0.012	0.005	0.009
β_{HL}	**FORM**	2.03	2.66	2.82	2.78	2.77	2.6

Table 2: Results of the reliability analysis for different radius of gyration, R_g

		G_{R1}	G_{R2}	G_{R3}	G_{R4}	G_{R5}
	R_g**(m)**	**<3400**	**3400-600**	**3600-3800**	**3800-4000**	**>4000**
P_f	**MCs**	0.505	0.116	0.075	0.052	0.23
	LHs	0.525	0.141	0.059	0.056	0.213
β_{HL}	**FORM**	2.16	3.48	3.78	4	2.62

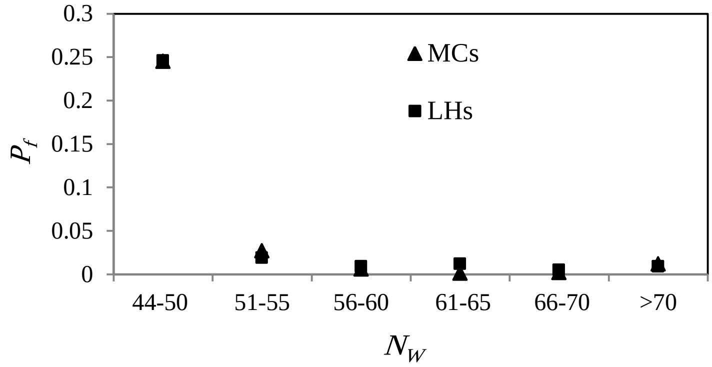

Figure 5: Probability of failure, P_f for different values of N_W

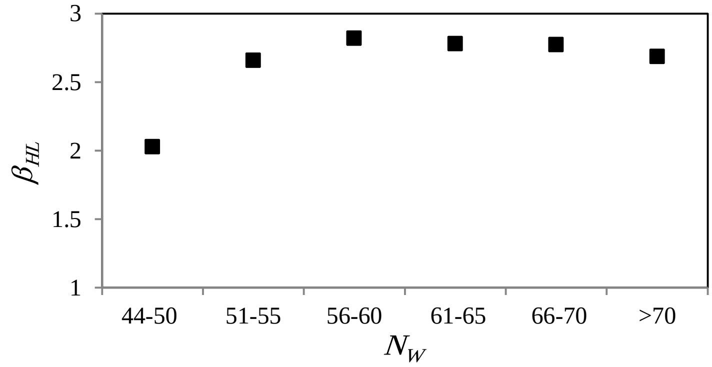

Figure 6: Reliability index, β_{HL} for different values of N_W

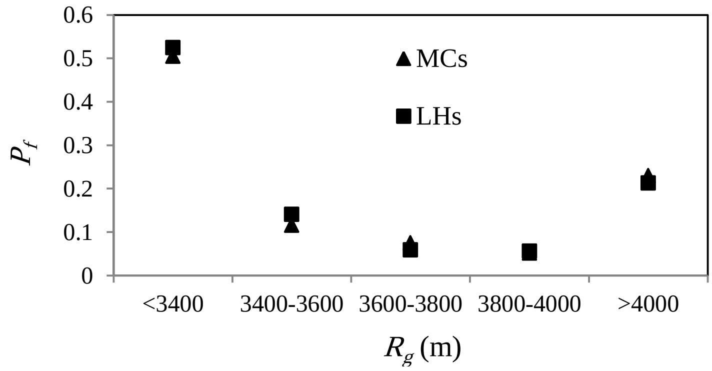

Figure 7: Probability of failure, P_f for different values of R_g

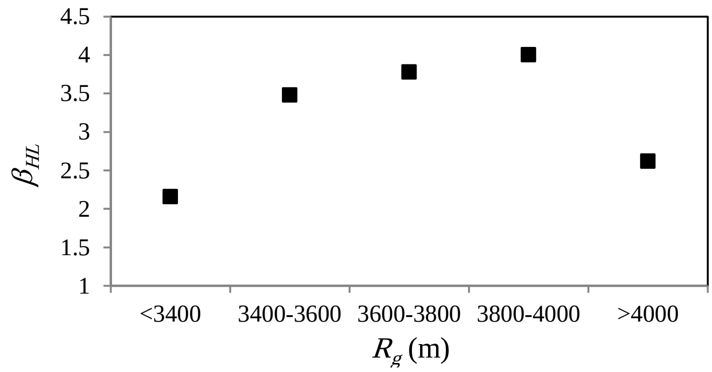

Figure 8: Reliability index, β_{HL} for different values of R_g

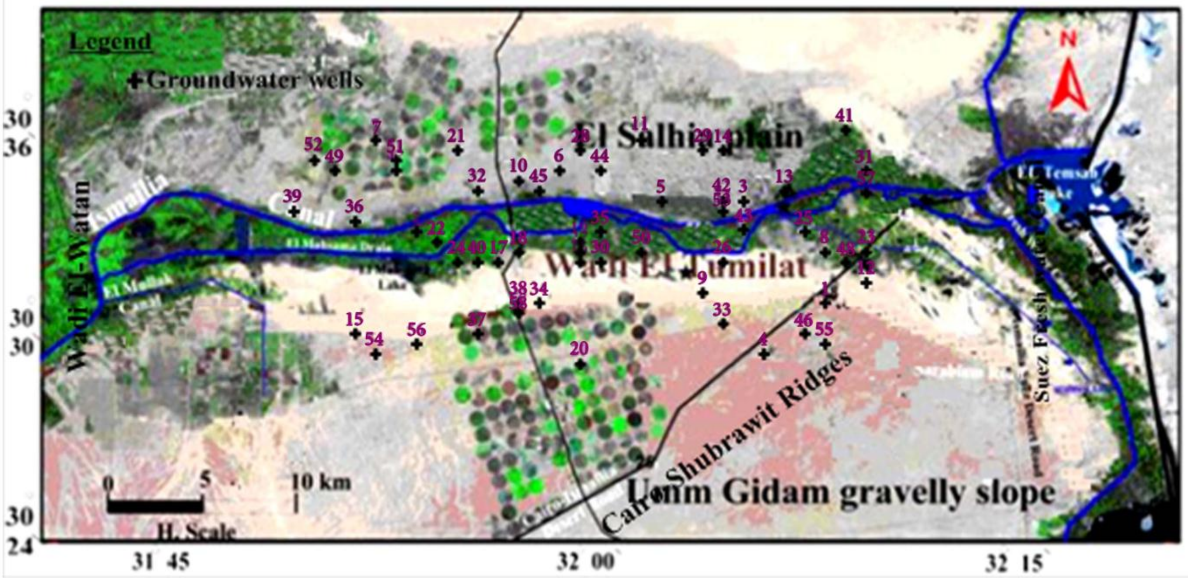

Figure 9: Optimal scheme corresponding to the design point

Table 3: Coordinates and pumping rates for the selected well system

Field Well No.	X-coordinate, m	Y-coordinate, m	Pumping rate, m^3/day	Field Well No.	X-coordinate, m	Y-coordinate, m	Pumping rate, m^3/day
1	27000	6000	28808	28	15000	13500	9473
2	7000	9500	28631	29	21000	13500	25756
3	23000	11000	28729	30	16000	8000	29387
4	24000	3500	22074	31	29000	12500	26797
5	19000	11000	24587	32	10000	11500	24935
6	14000	12500	14808	33	22000	5000	28099
7	5000	14000	28296	34	13000	6000	26369
8	27000	8500	29735	35	16000	9500	24719
9	21000	6500	21984	36	4000	10000	12536
10	12000	12000	7414	37	10000	4500	12589
11	18000	14000	23633	38	12000	6000	26132
12	29000	7000	23095	39	1000	10500	28520
13	25000	11000	25157	40	10000	8000	13362
14	22000	13500	29327	41	28000	14500	28425
15	4000	4500	10764	42	22000	11000	23958
16	15000	8000	3040	43	23000	10000	26468
17	11000	8000	25651	44	16000	12500	25611
18	12000	8500	28993	45	13000	11500	14203
19	15000	9000	22573	46	26000	4500	29530
20	15000	3000	8448	47	6000	12500	22192
21	9000	13500	2999	48	28000	8000	26975
22	8000	9000	29282	49	3000	12500	10384
23	29000	8500	15085	50	18000	8500	20018
24	9000	8000	28440	51	6000	13000	29029
25	26000	9500	23716	52	2000	13000	20174
26	22000	8000	19995	53	22000	10500	24800
27	15000	8500	23711	54	5000	3500	29690

7. Conclusions

In this research, a new methodology is suggested and applied for optimal management of groundwater in unconfined aquifers under uncertainty of hydraulic conductivity values. Two main consecutive approaches are presented. In the first approach, the FEM-GA model is developed to maximize the total pumping rate and minimize the number of wells, a surrogate of initial cost, by identifying the optimal location and discharge of wells in addition to the number of wells. The solution is repeatedly carried out, corresponding to each generated realization of the hydraulic conductivity values, to obtain numerous Pareto fronts. A unique Pareto-compromise solution for each obtained Pareto-front is determined and the corresponding state function is estimated. In the second approach, the obtained compromise solutions are divided into groups according to both the number of wells N_W and the radius of gyration R_g. Then, Monte Carlo simulation, Latin Hypercube sampling and First Order Reliability Method are applied to study the reliability of the estimated function corresponding to each realization. The results of the reliability analysis methods are found to be similar when applied on the case study of Quaternary aquifer of wadi El-Tumilat, Egypt. These results indicate that, N_W more than 55 reduce the probability of failure P_f to be less than 1%. On the other hand, R_g in the range from 3800 to 4000 gives the minimum P_f. In general, applying the proposed methodology on the case study showed its ability to help the decision maker to select the best operation conditions. The selected design point (or compromise solution of high level of reliability) is found having objectives (f_1 = 1260916 and f_2 = 58). This obtained point helps the decision maker to choose a single solution. Also, the number of wells, their discharges and their locations, corresponding to the design point, are determined.

Acknowledgements

The authors would like to express their deepest thanks to ***prof. Dr. Mohamed Gad***, Desert Research Centre, for his help for obtaining the real data of Quaternary aquifer of wadi El-Tumilat, Egypt.

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