

7-12-2020

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Recommended Citation

Mahanna, Hani; Fouad, Moharram; Radwan, Kamal; and Elgamal, Hoda (2020) "Modeling of Head Loss through Deep Bed Sand Filters.," *Mansoura Engineering Journal*: Vol. 40 : Iss. 4 , Article 21.
Available at: <https://doi.org/10.21608/bfemu.2020.102444>

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Modeling of Head Loss through Deep Bed Sand Filters

نمذجة الفواقد في الضغط خلال المرشحات الرملية العميقة

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المخلص

في محطات مياه الشرب تعتبر عملية الترشيح من اهم عمليات تنقية مياه الشرب ويتم فيها امرار المياه خلال طبقة مسامية لإزالة المواد العالقة والشوائب الموجودة بالمياه. في بداية عملية الترشيح تكون قيمة الفاقد في الضغط بسيطه ويمكن حسابها بواسطة معادلات تقريبية وتزداد هذه القيمة مع مرور الوقت حيث يتم انسداد في فراغات المرشح. تم تجهيز نموذج معلمي لمرشح رملي بمعمل الهندسة الصحية بكلية الهندسة جامعة المنصورة. الظروف التشغيلية للمرشح كانت خمسة متغيرات رئيسية هي عمق المرشح ومعدل الترشيح وفترة التشغيل وكمية المياه الداخلة وجرعة الشبة. تم استخدام عمق المرشح يتراوح من 80 سم الي 140 سم. كذلك تم استخدام جرعات مختلفة من الشبه تتراوح من 20 الي 50 مجم/لتر. معدل الترشيح المستخدم يتراوح من 4 متر/ساعة الي 8 متر/ساعة. ودرجات عكارة من 10 حتي 50 NTU. تم الحصول علي نموذج حسابي لفواقد الضغط مع ظروف التشغيل المختلفة (عمق المرشح ومعدل الترشيح و جرعة الشبه المضافه و زمن التشغيل ودرجة عكارة الأولية). من خلال هذا البحث تم الوصول الي معامل ارتباط قوي ($R^2 = 0,88$) للنموذج الحسابي المقترح. يبين النموذج المقترح للفاقد في الضغط ان اهم المتغيرات هي فترة الترشيح ومعدل الترشيح. النموذج البسيط المقترح يمكن استخدامه بسهولة وفاعلية كأداة قوية للتنبؤ بفترة تشغيل المرشح.

Abstract

Filtration is the main process in water treatment plant. In this process the water passing through some porous media (sand) to remove the suspended solids and impurities. In the beginning of filtration process, the head loss is small and it can be easily calculated by different empirical equations, but as the filter bed gets clogged, the head loss increases. The pilot plant was installed in sanitary engineering laboratory, Mansoura University. The operating conditions have five explanatory parameters. These parameters are filter depth, filtration rate, run time, influent turbidity, and alum dose. The filter depth was ranged from 80 to 140 cm and alum dose were ranged from 20 to 50 mg/lit. The rate of filtration was used in the range from 4 to 8 m/hr and the initial turbidities varied from 10 to 50 NTU. A mathematical model was obtained for head loss through deep bed sand filter with various operating conditions (filter depth, filtration rate, alum dose, run time, and initial turbidity). The proposed model yield highly accurate results with correlation coefficient (R^2) of 0.88. The proposed model showed that the most significant parameters on predicted head loss are the run time and filtration rate. Also, the simple proposed model can be easily and effectively used as a decision supporting tool for prediction of filtration run length.

Key Words

Deep bed filters, Head Loss, Filtration Rate

Introduction

The filtration process used in water treatment involves flow through a bed of porous media, such as sand, anthracite,

granite and activated carbon [1]. As the water passes through the media, the suspended particles are entrapped in the pore spaces of the media and thus removed from the water stream [2], [3].

The removal mechanism of particles by deep bed filtration is extremely complex and depends on the physical and chemical water characteristics, particles and media of filter. Particles removal results from a combination of transport, attachment and detachment mechanisms [4], [5].

The above events lead to head loss through filter media. In the beginning of filtration run the media grains are clean and the head loss is small, but as the media grains get clogged thus the resistance in the filter bed and the loss of head increases. In addition, the rate of filtration becomes very low so the filter media requires being backwashed [6], [7].

H. Mahanna et al. study the performance of deep bed sand filters under various operational conditions. In this research, a new predictive model for turbidity removal by deep sand filter was developed [8].

Head loss in clean filter

Several empirical equations have been developed, which are used to predict clean filter head loss. Some of the more commonly used equations are provided as the following: [9], [10], [11]

Carmen-Kozeny

$$\frac{f}{\phi} \frac{1-e}{e^3} \frac{L}{d} \frac{v^2}{g} \quad (1) \quad h_L =$$

Fair-Hatch

$$K^2 v S^2 \frac{(1-e)^2}{e^3} \frac{L}{d^2} \frac{v}{g} \quad (2) \quad h_L =$$

Hazen

$$\frac{1}{C} \frac{5.2 * 10^6}{T + 10} \frac{L}{d_{10}^2} v \quad (3) \quad h_L =$$

Rose

$$\frac{1.067}{\phi} C d \frac{1}{e^4} \frac{L}{d} \frac{v^2}{g} \quad (4) \quad h_L =$$

Ergun (modified Carmen-Kozeny)

$$h_L = \frac{f}{\phi} \frac{1-e}{e^3} \frac{L}{d} \frac{v^2}{g} + \frac{K_k}{\rho_w} \frac{(1-e)^2}{e^3} \frac{L}{d^2} \frac{v \mu_w}{g} \quad (5)$$

The above equations as stated in references [9],[10],[11] are used to determine the initial head loss where the filter media is clean.

There is another equation that used to predict head loss as a function of time. This equation is called Gregory equation and its form as the following [12],

$$h_L + \frac{K v C_o t}{(1-e)} \quad (6) \quad h =$$

Where:

$$f = 150 * \frac{1-e}{N_R} + 1.75 \quad (7)$$

$$N_R = \frac{d * v}{\mu_w} \rho_w \quad (8)$$

$$C_d = \frac{24}{N_R} + \frac{3}{\sqrt{N_R}} + 0.34 \quad (9)$$

Where:

h_L = initial head loss in m,

f = friction factor,

e = initial porosity,

L = media depth in m,

v = filtration velocity in m/s,

ϕ = particle shape factor (0.85 to 1.0), in eqns. (1), (4) and (5),

d = media grain diameter in mm,

g = gravity acceleration (9.81m/s²),

k = filtration constant,

S = shape factor (6.0 to 8.5), in eqn. (2),

ν = kinematic viscosity in m²/s,

C = coefficient of compactness (600-1200), in eqn.(3)

T = temperature °F,

d_{10} = media effective size in mm,

C_d = coefficient of drag (from eqn. (9)),

μ_w = absolute viscosity in Ns/m²,

ρ_w = density of water in kg/m³,

C_o = concentration of substance in fluid that lead to head loss,

k_k = Kozeny factor,

t = time in minutes,

N_R = Reynolds number

S. Han et al. developed a mathematical model of head loss through filter media in

rapid gravity filtration. This model could be expressed in equation (10) [13].

$$h = J_0 \int_0^L \left(1 + \frac{\gamma_0}{\rho_p} u^{-0.55} \sigma\right)^2 dz \quad (10)$$

Where:

h = Head loss value (m)

J_0 = the hydraulic gradient in the clean media bed. It can be calculated by the Carman-Kozeny equation or from experimental data.

γ_0 = constant for a specific filtration system.

L = Depth of filter media (m)

ρ_p = particles density (kg/m^3)

γ_0 = constant

u = superficial velocity (m/s)

σ = specific deposit (mg/L)

z = position in the filter bed (m)

H. Banejad, et al. produced a study about evaluation of head loss and iron removal by rapid sand filter. In this study by arrangement of modified Carman Kozeny, Rose and Gregory equation the time that head loss in sand media achieve to premises value, estimated. It has been reported that increasing in discharge and decreasing in inlet solids concentration, estimating time to given head loss increased [14].

Ç. Mehmet et al. conducted a neuro-fuzzy model to estimate head loss in dirty sand filters. Hydraulic loading rate, influent iron concentration, bed porosity, and operating time were selected as input variables. The fit between experimental results and model outputs was excellent, with correlation coefficient (R^2) greater than 0.99 [15].

A. Jusoh et al. used burned oil palm shell (BOPS) as a granular media in rapid sand filter. His results about the initial head loss (BOPS) media showed a good agreement with modified Kozeny-Carmen equation (Ergun equation). In this research the initial head loss for (BOPS) media was low compared to sand media with the same depth and velocity [10].

Hao L. et al. showed that both the Kozny and Ergun equations had limitations

in clean bed head loss predictions in crump rubber media [16].

The main objective of this research is analyze the factors affecting the run length for deep bed sand filters and to develop a simple predictive model for predicting head loss at any time through deep bed sand filters operation. This model is expected to help in the design of water and wastewater treatment units as well as predicting the run length for filtration by using different parameters.

Materials and Methods

For this study a Pilot Plant was designed and constructed where it allowed monitoring the media and water levels during the operations. The pilot plant was installed in the laboratory of sanitary engineering in the faculty of engineering – El- Mansoura University.

The experimental pilot plant, as shown in figure 1, consisted of the following main parts:-

Feeding tanks: the synthetic turbid water was prepared in four plastic feeding tanks. The four tanks divided into two groups each group consist of two tanks. The capacity of each tank was about 250 liters.

Feeding pump: The synthetic turbid water was transported from feeding tanks to a constant head tank by feeding pump. The horsepower of pump were 0.45 HP (Discharge $0.45 \text{ m}^3/\text{hr}$).

Constant head tank: The raw water has been fed from the feeding tanks to the sand filter via a constant head tank. The capacity of constant head tank was 45 liters and has dimensions of $30 * 30 * 50 \text{ cm}$. Constant head tank has confirmed a constant discharge to the plant whatever the difference in the water levels before and after it.

Filtration column: The major part of the pilot plant was the filtration column. The filtration column was made from galvanized steel with height 2.5 meters. The column consists of two parts each part 1.25 m height and it has square cross

section (20 *20 cm). The filtration column has one glass face to allow monitoring what is inside the column.

Backwash Pumps: two pumps were used for backwashing. The horsepower of each pump were 0.45 HP (Discharge 0.45 m³/hr).

pizometers: tubes installed behind filtration column to determine the head loss through the filtration media depth at different times (see Figure 3) **.Fittings and Plastic connecting pipes.**

Filter media

In this study, the filtration media was the sand media rested on the gravel layer. The sand depth in the filtration column was varied from 80 to 140 cm.

The gravel depth under the sand was about 20 cm with different sizes.

The gravel which is used has size from 2.0 mm to 9.0 mm. The gravel layer is located immediately below the filter sand media. The purpose of the gravel layer is to separate the filter media from the under drain system, to prevent media particles from clogging the under drain orifice, and to dissipate the backwash water jets from the orifice of the under drain system [17], [18].

The sieve analysis of the used sand media was done in Roads and airports engineering laboratory. From sieve analysis, the characteristics for the sand media were measured and summarized as shown in table 1.

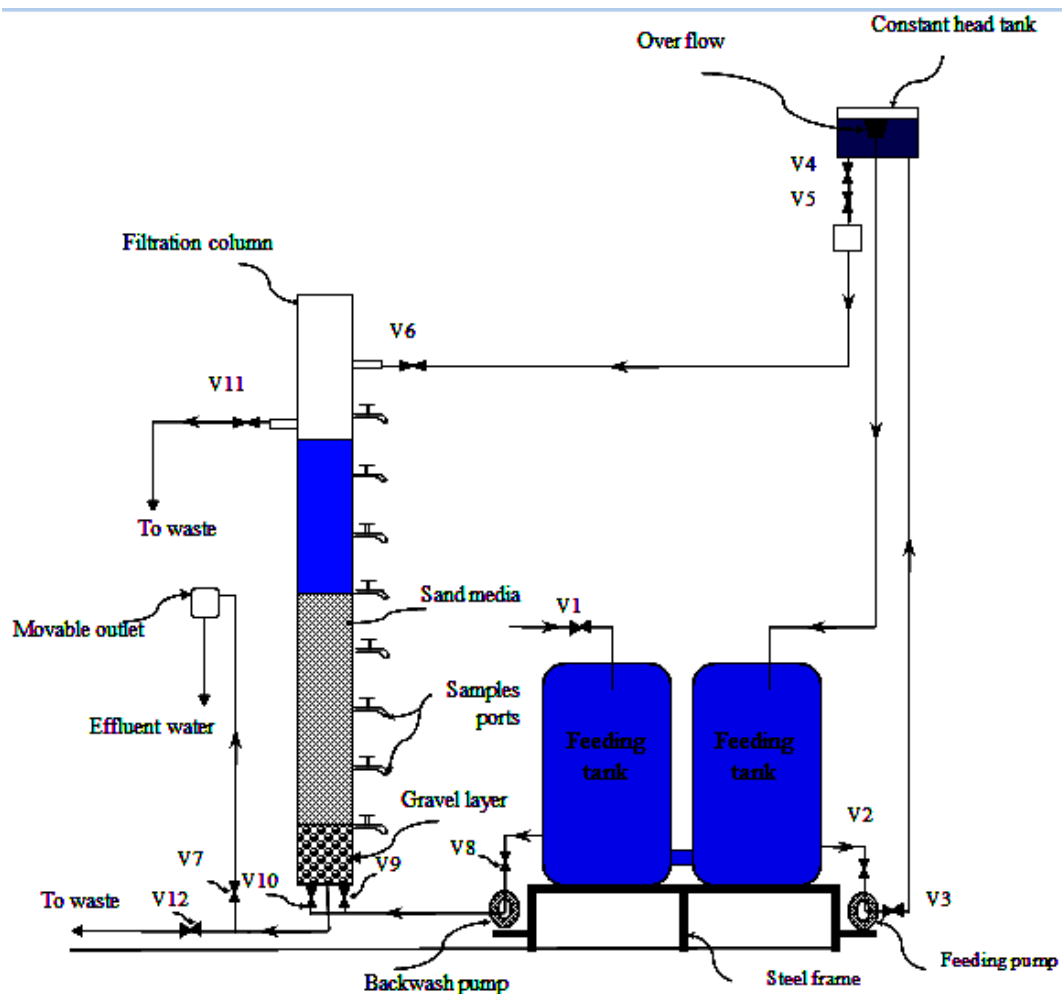


Figure 1: The schematic diagram for the experimental setup

Table 1: The Characteristic of the Used Sand Media

Sand depth, cm	Sand effective size, mm	Dry Density, gm/cm ³	Porosity, %	Specific gravity	Uniformity coefficient
80-140	0.72	1.65	37	2.55	2.15

Where: *Effective size* = D_{10} , *Uniformity coefficient* = D_{60}/D_{10}

To conduct the experimental work it is required to use some of equipments and carry out some tests. The turbidity was measured by using turbidimeter in nephelometric turbidity unit (NTU). Its model is Orbeco TB300-IR Lab Turbidimeter as shown in Figure 2.

By using fine clay soil and tap water it was able to achieve the synthetic raw water which can be used in the experimental work. The raw water was prepared by dispersing fine clay, passing from sieve No.200 having a size 0.074 mm in tap water.

This study has been extended to cover the different operation conditions for the sand bed filter. The alum doses which were added ranged from 20 to 50 mg/lit.



Figure 2: Turbidimeter model (Orbeco TB300-IR)

Further, the rates of filtration were 4, 5, 6, and 8 m/hr. In addition, the filter depth was used 80, 100, 120, and 140 cm. The tests were done using different influent turbidities, which ranged from 10 to 50 NTU.

Pilot Plant Operation Modes

The operation of the pilot plant was controlled by 12 valves. These valves facilitate different modes of filter operation.

- 1- Filtration operation mode.
- 2- Backwash operation mode.

The filtration rate was controlled by valves V4 and V5. During filtration mode, Valve V4 was completely opened and valve V5 was opened gradually and the time for collecting certain volume of water was measured to estimate discharge. Moreover, Valve V4 was fully closed during backwash mode.

Samples Collection and Demonstration

The design of the pilot plant allowed the monitoring and measuring the water quality through different depths of media length and the head loss cross the filter bed. The head loss was measured by pizometers tubes fixed at the top and bottom levels of media depth as shown in Figure 3.

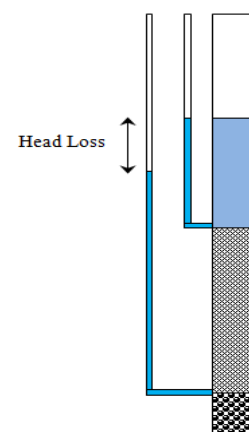


Figure 3: Head loss measurement by pizometers tubes.

Through the length of the filtration column 9 connection points were fixed each 20 cm as samples points. The pilot plant also, allowed taking sample from influent and effluent water simply.

Results and Analysis

From the experimental results about head loss, the length of run was influenced by alum dose, influent Turbidity, filtration rate, and depth of sand bed. Figures 4,5, and 6 show the effect of filtration rate and alum dose on run length for 80 cm,100 cm, and 120 cm filter depth respectively in case of influent turbidity equal 30 NTU. However, Figures 7,8, and 9 show the effect of filtration rate and alum dose on run length for 80 cm,100 cm, and 120 cm filter depth respectively in case of influent turbidity of 20 NTU.

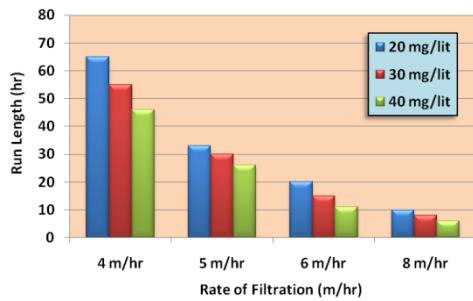


Figure 4: Run length versus filtration rate for 80 cm depth and $C_o=30NTU$.

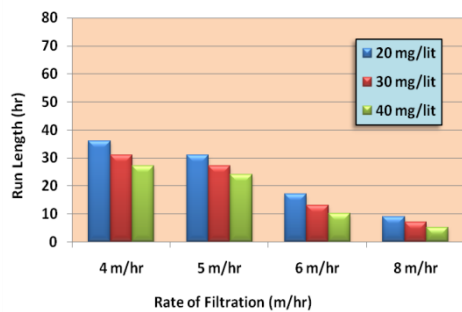


Figure 5: Run length versus filtration rate for 100 cm depth and $C_o=30NTU$.

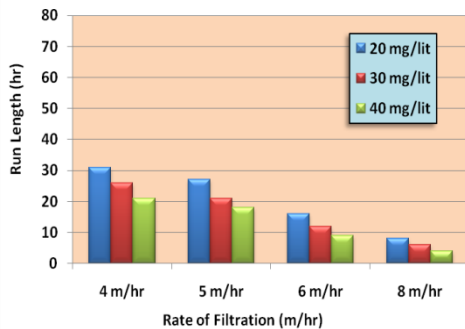


Figure 6: Run length versus filtration rate for 120 cm depth and $C_o=30NTU$.

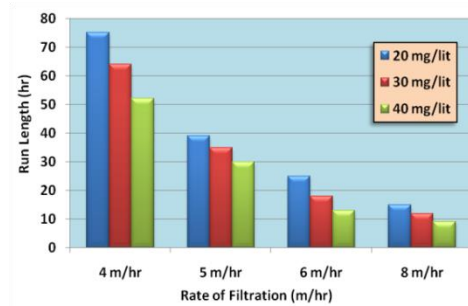


Figure 7: Run length versus filtration rate for 80 cm depth and $C_o=20NTU$.

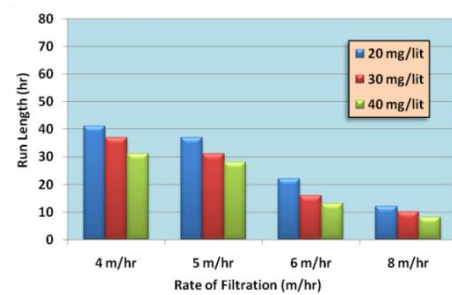


Figure 8: Run length versus filtration rate for 100 cm depth and $C_o=20NTU$.

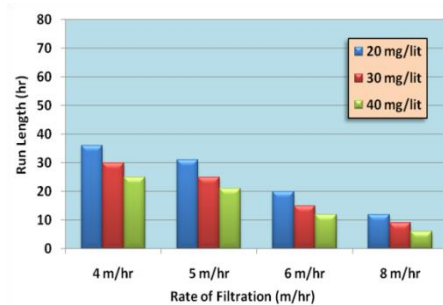


Figure 9: Run length versus filtration rate for 120 cm depth and $C_o=20NTU$.

From above figures, it was noticed that, depth of filter and filtration rate have significant effect on run length, but the alum dose and Influent turbidity have less effect.

The measured data was used to develop a simple predictive model for head loss through sand filters. Regression analysis using solver function in Excel software program was used for the model development.

After many trials, the following model which yielded the highest coefficient of determination (R^2 of 0.88)

and lowest percent of S_e/S_y of 0.347 (S_e is the standard error of predicted head loss and S_y is the standard deviation of measured head loss)

$$H_L = 2.47 T + 0.43 L + 13.22 V_f + 0.45 S + 0.82 C_o - 114.3$$

Where:

H_L = Predicted head loss (cm)

T = Run time (hr)

L = Filter depth (cm)

V_f = Filtration rate (m/hr)

S = Alum dose (mg/lit)

C_o = Influent Turbidity (NTU)

The relationship between the predicted and measured head loss is presented in Figure 10. The goodness of fit statistics is also shown in the figure.

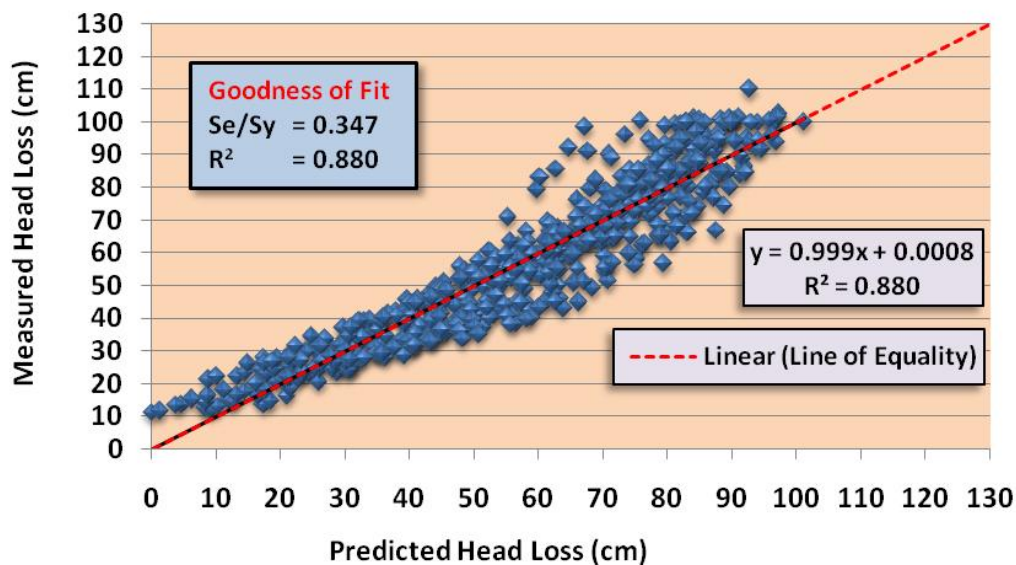


Figure 10: Measured versus Predicted Head Loss

Model Precision and Bias

Figure 8 and the goodness of fit statistics of the model show very low scatter and highly accurate predictions. Bias is defined as the systemic difference between observed and predicted values. The bias in the model predictions was evaluated statistically. A linear regression on the measured and predicted head loss was performed and the following hypothesis tests at a significance level of 5 percent ($\alpha = 0.05$) were done.

Hypothesis 1: Determines whether the linear regression model developed using measured and predicted head loss has an intercept of zero by testing the following null and alternative hypotheses:

H_o : Model intercept = 0; and

H_A : Model intercepts $\neq 0$.

A rejection of the null hypothesis (p -value < 0.05) would indicate the linear model had an intercept significantly different from zero at the 5 percent level of significance. This means biased model predictions.

Hypothesis 2: Determines whether the linear regression model developed using measured and predicted head loss has a slope of unity by testing the following null and alternative hypotheses:

H_o : Model slope = 1.0; and

H_A : Model intercepts $\neq 1.0$.

A rejection of the null hypothesis (p -value < 0.05) would involve that the linear model has a slope significantly different from 1.0 at the 5 percent level of significance and thus the model systemically yields biased predictions.

Hypothesis 3: A paired t-test was done to determine whether the measured and predicted head loss had the same average.

H_0 : Mean measured head loss = Mean predicted head loss; and

H_A : Mean measured head loss \neq Mean predicted head loss.

A rejection of any of the three null hypotheses (p -value < 0.05) would imply that predicted head loss model results are biased predictions. If the model passed all three hypotheses tests successfully, the model predictions are not biased. The results of the conducted hypotheses tests are summarized in Table 2.

Table 2 Statistical Comparison of Measured and Predicted head loss Data

Hypotheses	df*	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
(1) H_0 : Intercept=0	1	0.000808	0.946772	0.000854	0.999	-1.85872	1.860341
(2) H_0 : slope = 1.0	1	0.999987	0.015324	65.25757	0.999	0.96989	1.030083
(3) H_0 : Mean Measured = Mean Predicted head loss	580	-	-	-	0.999	-	-

*df = degrees of freedom

Sensitivity Analysis

The predicted model was used to test the sensitivity of predicted head loss to each parameter. The results of the sensitivity analysis are shown in Figures 11 to 15.

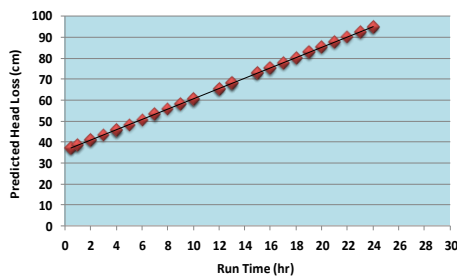


Figure 11: Run Time versus Predicted Head Loss

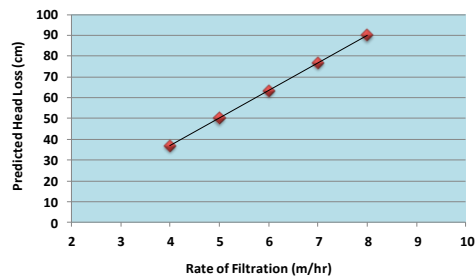


Figure 12: Filtration Rate versus Predicted Head Loss

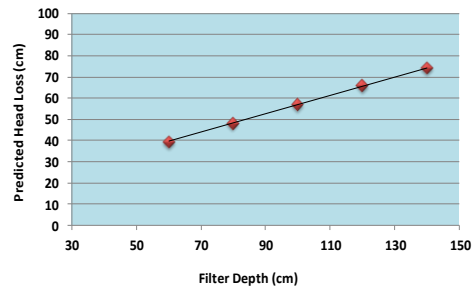


Figure 13: Filter Depth versus Predicted Head Loss

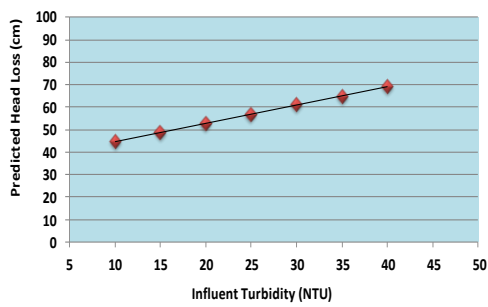


Figure 14: Influent Turbidity versus Predicted Head Loss

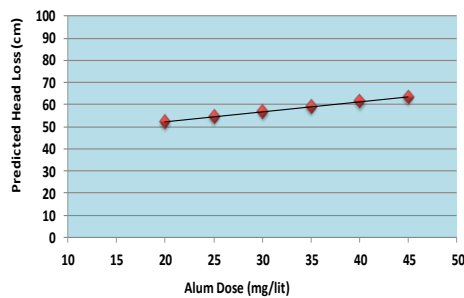


Figure 15: Alum Dose versus Predicted Head Loss

From the above figures, the sensitivity results show that, Run time increasing has significant effect on head loss .It was also noticed that, as the filtration rate and filter depth increase, the predicted head loss significantly increase. Finally, as the alum dose increase, the head loss slightly increases.

Initial Head loss by Expected Model and Other Common Equations

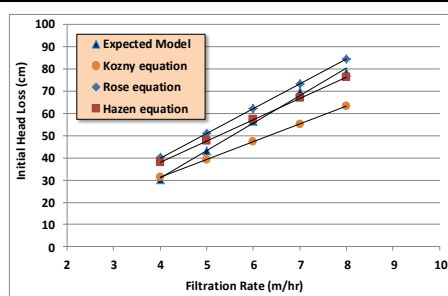


Figure 16: Initial Head Loss versus Filtration Rate at Depth 120 cm by Different Equations.

Figure 16 showed that the head loss value from expected model is close to Kozny equation value at low filtration rate. While, at high rates, the expected model values tend to Hazen equation values.

Model Restrictions

It is obvious that, the model has been deduced based on regression analysis of experimental data, so the model will be of use in the range at which the data were taken. The suggested conditions for applying the model are,

- Filtration rate < 200 m³/m²/day
 - turbidity level < 50 NTU
 - temperature < 40 °C
 - alum dose < 50 mg/L
 - particle size of media = 0.7-1.0 mm
- any limits of parameters out of range must be studied then the model can be modified.

Summary and Conclusions

The present study was conducted using pilot plant which installed in Sanitary engineering laboratory in faculty of engineering, Mansoura University. Head loss through deep bed sand filter was measured in various runs .Based on the measured data, a simple head loss predictive model for deep bed sand filters was developed. This model predicted the head loss as a function of Run time, rate of filtration, filter depth, Influent turbidity, and alum dose. The model showed excellent prediction accuracy with R² of 0.88 and S_e/S_y of 0.347. The results of the conducted hypotheses tests showed that the model predictions are not biased. The sensitivity study of the model identified that Run time and filtration rate as key factors affecting the predicted head loss

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