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Scour Downstream Sudden Expansion Stilling Basin
النحر خلف أحواض التهدئة فجائية الاتساع

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الخلاصة
النحر الحادث نتيجة القفزه الهيدروليكيه الحره خلف المنشآت الهيدروليكيه يؤدى الى إنهيار هذه المنشآت و لذلك التحكم والسيطره على هذه الظاهره يمثل أهميه قصوى. الهدف الرئيسى من هذه الدراسه هو تقليل خصاص حفرة النحر الحادث خلف احواض التهدئه ذات الاتساع المفاجئ. تم إجراء دراسه معمليه لدراسة تأثير نسبة الإتساع ومكان العتب. حيث أجريت جميع التجارب (90 تجربه) تحت ظروف سريان متماثله فى قناه معمليه باستخدام عده تصرفات وعدد من فتحات البوابه بحيث تراوح رقم فرود من 3.42 الى 8.67. تم دراسة خمس قيم مختلفه من نسبة الاتساع لحوض التهدئة (e (= 2.73, 1.92, 1.76, 1.50 and 1.25 و أربع قيم مختلفه لموضع العتبه (L_x/L_B = 0.20, 0.30, 0.40 and 0.50). وقد تم استخدام نظرية التحليل البعدى لربط المتغيرات المختلفه التى تؤثر على خصائص حفره النحر خلف الاحواض فجائيه الاتساع ووضعها فى صوره لابعديه. وقد تبين أن نمط السريان فى حوض التهدئه غير متماثل فى معظم الحالات مما أدى الى نحر وترسيب غير متماثل أيضا, وتزداد خصائص حفره النحر (عمق و طول الحفره) وكذلك مقدار الطاقه المشتته بزياده رقم فرود. حيث وجد ان افضل اتساع نسبى هو 1.50 وافضل مكان نسبى للعتب على بعد 0.30 من طول الفرشه من فتحه البوابه. تم استباط معدلات تجريبيه يمكن استخدامها للتنبؤ بأبعاد حفره النحر فى الحالات المشابهه لحاله الدراسه.

Abstract
Local scour due to free hydraulic jump downstream hydraulic structures may cause damage or complete failure of these structures, so controlling of this phenomenon is very important. The main goal of this study is to reduce the characteristics of a scour hole downstream sudden expansion stilling basin. An experimental study was conducted to study the effect of expansion ratio and position of the sill. Ninety experimental runs were carried out considering the wide range of Froude numbers ranging from 3.42 to 8.67. Five values of the expansion ratio (e = 2.73, 1.92, 1.76, 1.50 and 1.25) and four values of the relative position of lateral single sill (L_x/L_B = 0.20, 0.30, 0.40 and 0.50) were investigated. The dimensional analysis was employed to drive expressions correlating the different variables affecting the scour phenomena. It was found that, the flow patterns in most of the cases were a symmetrical and the resulting scour and deposition were also a symmetrical. The relative scour depth, the relative scour length and the relative energy loss, increase by increasing the initial Froude number and vice versa. The expansion ratio (e = 1.50) gives the minimum values of scour dimensions. The best location of the sill for reducing the scour dimensions at $0.30L_B$ from the gate opening. Prediction equations were developed using the multiple linear regression (MLR) to model the relative scour depth D_s/y_1 and the relative scour length L_s/y_1.

Keywords
Local Scour, Hydraulic jump, Stilling basins, Sudden Expansion, Lateral Single Sill.

1. Introduction
Scour is a natural phenomenon caused by the flow of water over an erodible boundary. Flow underneath gates is a tremendous amount of potential energy, which is converted into kinetic energy downstream the hydraulic structures. This energy should be dissipated to prevent the possibility of excessive scouring of the downstream river bed, minimize erosion and the undermining of the structures, which endanger the structure safety. Many studies take place to reduce maximum scour depth

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downstream hydraulic structure. Bremen and Hager, (1994) investigated the optimal configuration of the central baffle sill in symmetric sudden expanding stilling basins. El-Gamel, et al., (2002) studied the effect of using stilling basins on local scour phenomena. Negm, et al., (2002) investigated experimentally the effect of different expansion ratios of sudden expanding stilling basin on scour characteristics of downstream movable soil. Negm, et al., (2003) investigated experimentally the effect of using central sill at different positions and different heights on the scour characteristics downstream of abruptly enlarged stilling basins. Negm, (2007) investigated experimentally the effect of the position of central symmetric sill on the maximum scour depth downstream of radial stilling basin. Negm, (2004) studied the effect of sill arrangements in sudden expanding stilling basin on scour characteristics downstream of the basin. Saleh, et al., (2003) investigated experimentally the effect of using asymmetric side sill both single and double side sills on the maximum scour depth downstream of sudden expanding stilling basins. Saleh, et al., (2003) investigated experimentally the effect of using end sill on scour characteristics downstream of sudden expanding stilling basins. Helal, et al., (2013) studied the effect of the position of sills on the maximum scour depth downstream hydraulic structures. Negm, et al., (2008) used the curved deflector to reduce the maximum depth of scour downstream multi-vents regulators. Ibrahim and Negm, (2009) studied the effect of height of curved deflector wall on maximum scour depth downstream of multi-vents regulators. Abdel-Aal, et al., (2009) investigated experimentally the effect of the guide wall position on local scour downstream of stilling basins. Abdel-Aal, et al., (2008) investigated experimentally the effect of the symmetric side slopes of trapezoidal channel section started abruptly downstream the transition length. Fahmy, et al., (2012) studied the effect of the different shapes of corrugated beds on the characteristics of a hydraulic jump and downstream local scour. Bestawy, et al., (2013) studied the effect of the different shapes of a single line baffle piers on the characteristics of a hydraulic jump and downstream local scour. Helal, (2013) studied the effect of multi-lines of floor water jets on scour hole behind control structures. Imran and Akib, (2013) investigated the potential use of corrugated and roughened beds for reducing the hydraulic jump length and sequent depth. Helal, (2014) studied the effect of single line of floor water jets on scour hole downstream of hydraulic structures. Ahmed, et al., (2014) investigated experimentally the effect of using spaced triangle strip corrugated bed on submerged hydraulic jump characteristics. The present work aims to study the effect of expansion ratio and the position of the lateral single sill of certain shape and dimensions on the characteristics of a scour hole downstream sudden expansion stilling basin.

2. Dimensional Analysis

Dimensional analysis based on Buckingham theory was used to develop functional relationship between the maximum scour depth and the other variables as shown in Figure 1. The maximum scour depth, D_s, downstream of the stilling basins could be expressed as follows:

$$D_s = f(B, b, L_w, L_B, L_x, h_x, t_x, \alpha, G, H_u, y_1, y_2, y_t, v1, L_s, \rho, \rho_s, g, D50) \quad (1)$$

In which, B is the flume width, b is the gate opening width, L_w is the wing wall length, L_B is the apron length from the gate opening, L_x is The sill position measured from the gate opening, h_x is the sill height, t_x is the top width of the sill, α is the downstream angle of the sill, G is the gate opening height, H_u is the upstream water depth, y_1 is initial depth of hydraulic jump, y_2 is sequent depth of hydraulic jump, y_t is the tail water depth, v_1 is mean velocity at the initial depth, D_s is the maximum scour depth, L_s is maximum scour length, ρ is the density of water, ρ_s is the density of sand particles, g is the gravitational acceleration, D_{50} is the mean diameter of the sand base.

Applying the Buckingham theorem with ρ, y_1, v_1 as repeating variables, Equation Error! Reference source not found. can be written in dimensionless form as:

$$\frac{D_s}{y_1}, \frac{L_s}{y_1} = f(F_1, e, \frac{y_2}{y_1}, \frac{\Delta E}{E_1}, \frac{L_X}{L_B}) \qquad (2)$$

In which, D_s/y_1 is the relative maximum scour depth, L_s/y_1 is the relative maximum scour length, F_1 is the initial Froude number, e=B/b is the expansion ratio, y_2/y_1 is the relative depth of the hydraulic jump, $\Delta E/E_1$ is the relative energy loss and L_X/L_B is the relative position of the sill.

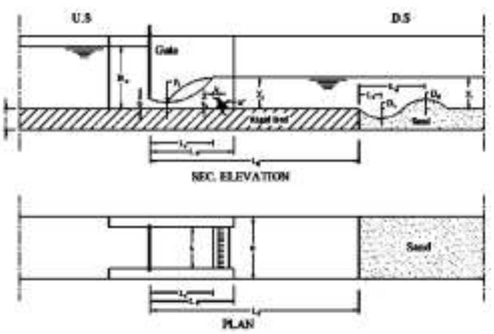

Figure 1 Definition sketch for the experimental model

3. Experimental Work

3.1. The Flume

Experiments were carried out in the Hydraulics Laboratory of the Faculty of Engineering, Zagazig University, Egypt, using a rectangular re-circulating adjustable flume of 30cm width, 46.8cm height and 15.6m length. The flume is equipped with a tailgate to control the tail water depth. A pre-calibrated orifice meter fixed in the feeding pipeline was used to measure the discharges. The tail-water depth of flow was controlled by a tailgate fixed at the end of the flume. The basin was made from a clear prespex to enable visual inspection of the phenomenon being under investigation. A general view of the flume shown in Photo 1.

3.2. The Experimental Models

The experimental model consisted of two abutments made from wood with a length of 60cm. The wood was painted very well by a water proof material (plastic) to prevent wood from changing its volume by absorbing water. A control sluice gate is made from Perspex of thickness 6mm and slide through two vertical grooves. The rigid bed thickness was 10cm and the model height over it equaled 35cm. The distance from the sluice gate to the end of the apron is 100cm. The sills were made from wood and fitted in floor body by using epoxy steel. A movable sand bed of length 2.0 m and 10cm thickness was formed just DS of stilling basin, Which was made of coarse sand passing through IS sieve opening 2.36 mm and retained on IS sieve opening 1.18mm. A point gauge is installed to measure the bed level and the water depth. The gauge is mounted on carriage moving in the flow and the perpendicular directions.

The width of the flume B is kept constant to 30cm, while the width of gate opening b is variable as 11, 15.6, 17, 20 and 24cm to obtain expansion ratios of e = 2.73, 1.92, 1.76, 1.50 and 1.25 respectively. The relative height of the sill h_x is kept constant to h_x/y_1=1.0. The relative top width of the sill t_x is kept constant to t_x/h_x=0.50. Downstream slope of the Sill is kept constant to 1:1. Different positions of the sill are considered such that L_X/L_B = 0.2, 0.3, 0.4 and 0.5. A general view of different sills shown in Photo 2. Range of discharges and gate openings were used such that the initial Froude number ranged from 3.42 to about 8.67. The total number of runs was about 90. Each run lasted about 60 minutes here about 85% of maximum scour occur.

3.3. The Experimental Procedures

The experimental procedure was started by leveling the sand bed surface by using a plate attached to the instrument carriage. The required gate opening height is adjusted. The pump was switched on and the required discharge was passed gradually using the discharge control valve. The tailgate was adjusted to form a free jump over the rigid bed. After the stability conditions are attained, the following measurements are taken, the upstream water depth, the initial water depth y_1, and the sequent water depth y_2. During each run the flow pattern was observed and sketched. After the required time (60 min.), the flume pump is stopped. The experiment is left, until the sand bed is completely dry. The scour mesh was measured, and the stilling basin model was changed and steps were repeated.

Photo 1 A general view of the flume

Photo 2 A general view of different Sill

4. Analysis And Discussions

4.1. Effect Of Expansion Ratio

The effect of different expansion ratio (e = 2.73, 1.92, 1.76, 1.50 and 1.25) on scour hole characteristics, have been investigated. The relationships between the initial Froude number F_1 and both of the maximum relative scour depth D_s/y_1, the maximum relative scour length L_s/y_1 and the relative energy loss DE/E_1 were shown in Figure 2, Figure 3 and Figure 4 respectively. It was found that, the relative scour depth, the relative scour length and the relative energy loss, increase by increasing the initial Froude number and vice versa.

The case of expansion ratio e = 2.73 gives the maximum energy loss DE/E_1 but not the best in the scour hole dimensions D_s/y_1 and L_s/y_1, this is due to the flow pattern, where the flow pattern is asymmetric. It is observed that the main jet of the flow inside the basin directed towards the right or left side of the basin. The maximum scour hole occurs in the same direction of the main jet and another smaller scour hole may be formed on the other side. The behavior of the main jet of flow in both cases of e = 2.73, 1.92 and 1.76 are mostly similar.

The expansion ratio e = 1.50 gives the minimum scour hole dimensions D_s/y_1 and L_s/y_1 but not the best in the energy loss DE/E_1, this is due to the flow pattern, where the flow pattern is almost symmetric and causing little scour depth. Flow and scour patterns for different expansion ratio shown in Figure 12.

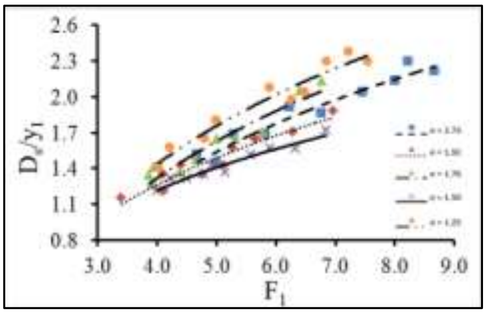

Figure 2 Relationship between D_s/y_1 and F_1 for different expansion ratios (e)

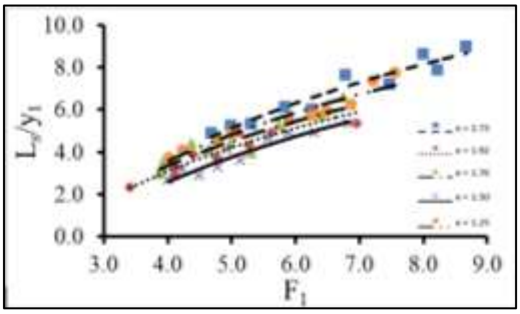

Figure 3 Relationship between L_s/y_1 and F_1 for different expansion ratios (e)

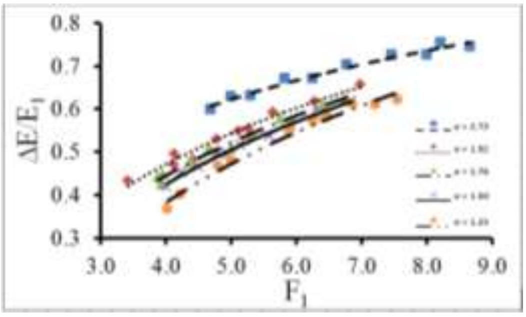

Figure 4 Relationship between F_1 and DE/E_1 for different expansion ratios (e)

4.2. Effect Of Lateral Single Sill Position (L_x/L_B) On Scour Hole Characteristics

To reduce the scour hole dimensions (maximum scour depth D_s and maximum scour length L_s) DS sudden expansion stilling basins, a new sill shape was tested in the present study. The effect of different Lateral Single Sill position (L_x/L_B = 0.20, 0.30, 0.40 and 0.50) on scour hole characteristics have been investigated. The relationships between the initial Froude number F_1 and both of the maximum relative scour depth D_s/y_1, the maximum relative scour length L_s/y_1 and the relative energy loss $\Delta E/E_1$ were shown in Figure 5, Figure 6 and Figure 7 respectively. It was found that, the relative scour depth, the relative scour length and the relative energy loss, increase by increasing the initial Froude number and vice versa. In the case of 0.20 and 0.30, the scour occurs in the middle, but in the case of 0.40 and 0.50 the scour occurs on the sides as shown in Figure 13. The relative position of the Lateral Single Sill (L_x/L_B = 0.30) gives the maximum energy loss and minimum relative scour length and depth.

The lateral single sill of L_x/L_B = 0.30 at F_1 = 5.0 recorded, scour depth reduction by about 23%, scour length reduction by about 19%, energy loss increased by about 8.5%, Table 1 shows the comparison between the results of different position of lateral single sill L_x/L_B at F_1 = 5.0.

Table 1 Comparison between the results of different position of lateral single sill L_x/L_B at F_1 = 5.0.

L_x/L_B	0.20	0.30	0.40	0.50
D_s/y_1	9%	23%	-29%	-68%
L_s/y_1	-48%	19%	-64%	-80%
$\Delta E/E_1$	6.6%	8.5%	4.4%	2.4%

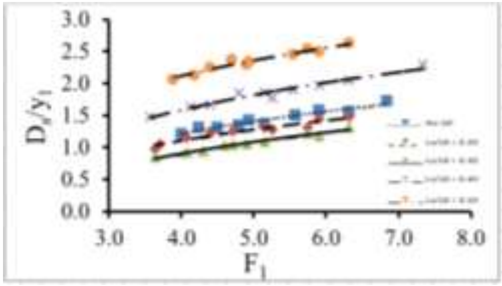

Figure 5 Relationship between D_s/y_1 and F_1 for different position of lateral single sill (L_x/L_B)

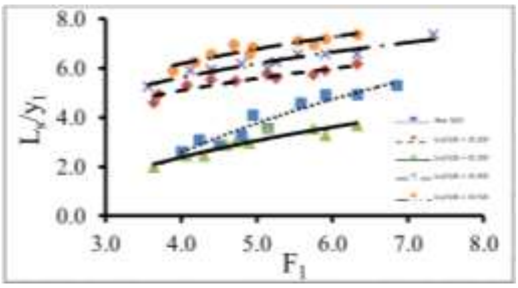

Figure 6 Relationship between L_s/y_1 and F_1 for different position of lateral single sill (L_x/L_B)

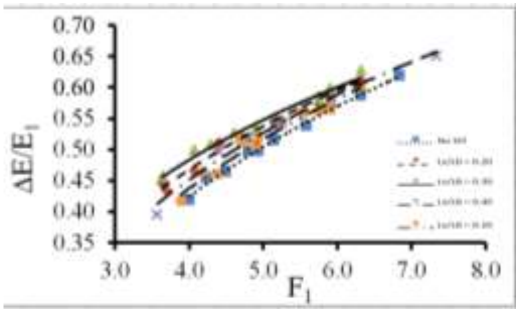

Figure 7 Relationship between F_1 and DE/E_1 for different position of lateral single sill (L_x/L_B)

5. Statistical Regression

The regression tool was used to carry out the necessary regression tasks and statistical analysis. With that tool and based on the experimental data, the statistical equations were proposed to predict the scour dimensions D_s/y_1 and L_s/y_1 downstream sudden expanding stilling basin with and without lateral single sill.

The scour dimensions D_s/y_1 and L_s/y_1 DS of SESB for no sill case could be estimate from the following equations (1) and (2).

$$\frac{D_s}{y_1} = 0.3289 + 0.0835e + 0.2015F_1 \quad (1)$$

$$\frac{L_s}{y_1} = -2.8196 + 1.1222e + 0.9911F_1 \quad (2)$$

The scour dimensions D_s/y_1 and L_s/y_1 DS of SESB for lateral single sill case could be estimate from the equations (3) and (4)

$$\frac{D_s}{y_1} = 0.331 + 0.893\frac{h_x}{y_1} - 4.671\frac{L_x}{L_B} + 0.133e + 0.181F_1 \quad (3)$$

$$\frac{L_s}{y_1} = -1.926 + 5.017\frac{h_x}{y_1} - 19.886\frac{L_x}{L_B} + 1.267e + 0.786F_1 \quad (4)$$

The regression statistics of Eqs. (1), (2), (3) and (4) are shown in Table 2.

Table 2 The regression statistics of Eqs. (1), (2), (3) and (4).

Regression Statistics	Eq. (1)	Eq. (2)	Eq. (3)	Eq. (4)
R Square	**0.884**	**0.939**	**0.918**	**0.898**
Adjusted R Square	**0.877**	**0.935**	**0.914**	**0.893**
Standard Error	**0.104**	**0.416**	**0.119**	**0.538**

6. Conclusions

The present study introduced the following results.

1- The flow patterns in most of the cases were asymmetric and the resulting scour and deposition were also asymmetric.

2- The relative scour depth, the relative scour length and the relative energy loss, increase by increasing the initial Froude number and vice versa.

3- The optimum expansion ratio was found to be e = 1.50.

4- The optimum location of the lateral single sill was found to be at L_x/L_B = 0.30 from the gate opening, where reduced the different scour dimensions D_s/y_1 and L_s/y_1 by about 23% and 19%, respectively.

5- An empirical equation was developed by regression analysis to predict the different scour dimensions DS of SESB with and without lateral single sill.

7. Notations

b	The gate opening width
B	The flume width
e	The expansion ratio
L_W	The wing wall length
L_B	The apron length from the gate opening
L_x	The sill position measured from the gate opening
h_x	The sill height
t_x	The top width of the sill
α	The downstream angle of the sill
H_u	The upstream water depth
G	The gate opening height
y_1	The initial depth of hydraulic jump
y_2	The sequent depth of hydraulic jump
y_t	The tail water depth
V_1	The mean velocity at the initial depth
F_1	The initial Froude number
E_1	Specific energy at the initial water depth of a hydraulic jump
E_2	Specific energy at the sequent water depth of a hydraulic jump
ΔE	Energy losses
D_s	The maximum scour depth
L_s	The maximum scour length
ρ	The density of water
ρ_s	The density of sand particles
g	The gravitational acceleration
D_{50}	The mean diameter of the sand base
SESB	Sudden expansion stilling basin

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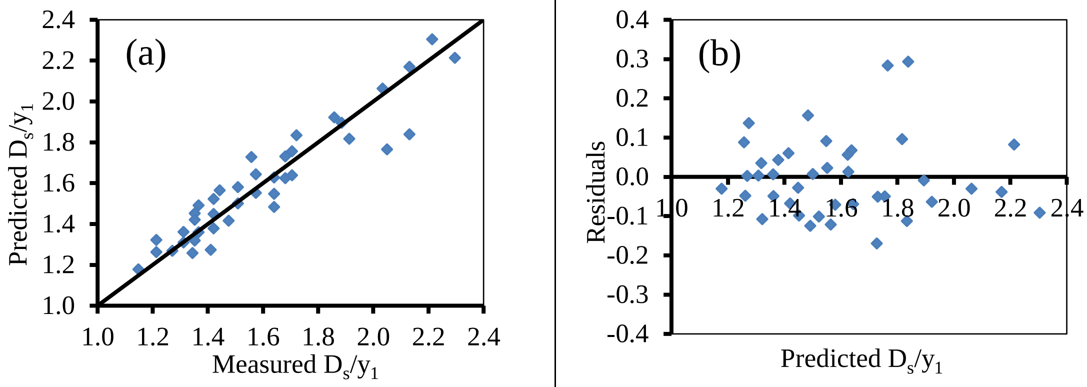

Figure 8 Comparison between the measured and predicted data for all experimental data of D_s/y_1 for Eq. (1)

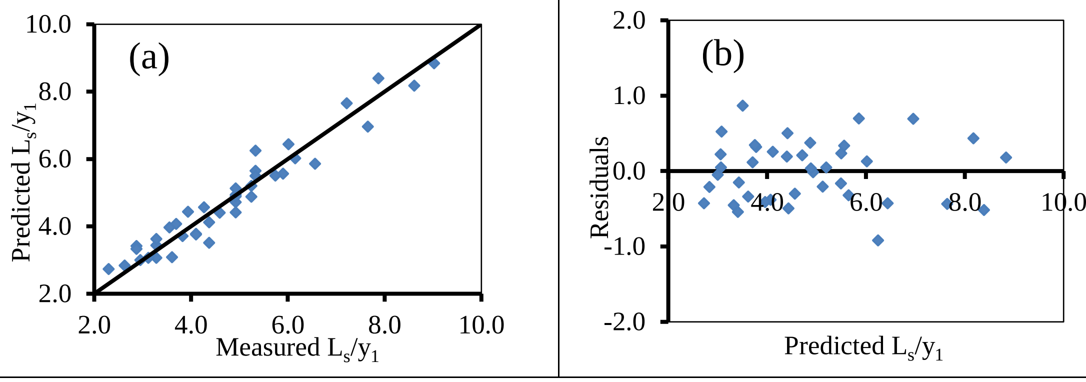

Figure 9 Comparison between the measured and predicted data for all experimental data of Ls/y1 for Eq. (2)

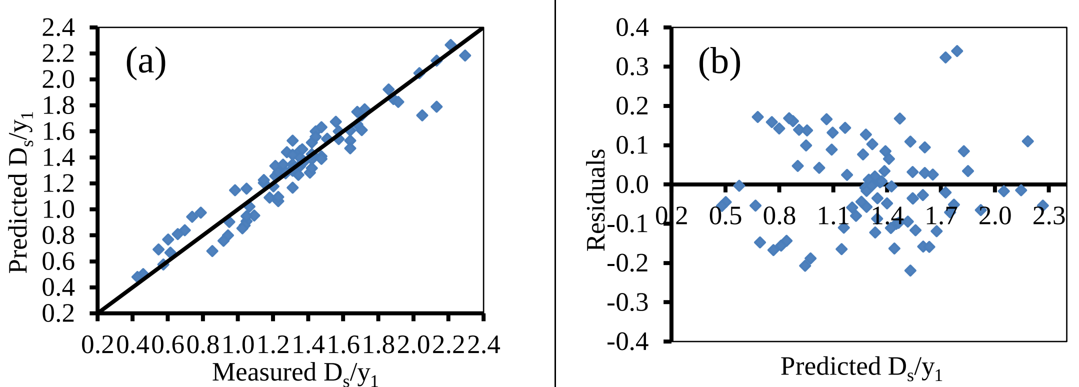

Figure 10 Comparison between the measured and predicted data for all experimental data of Ds/y1 for Eq. (3)

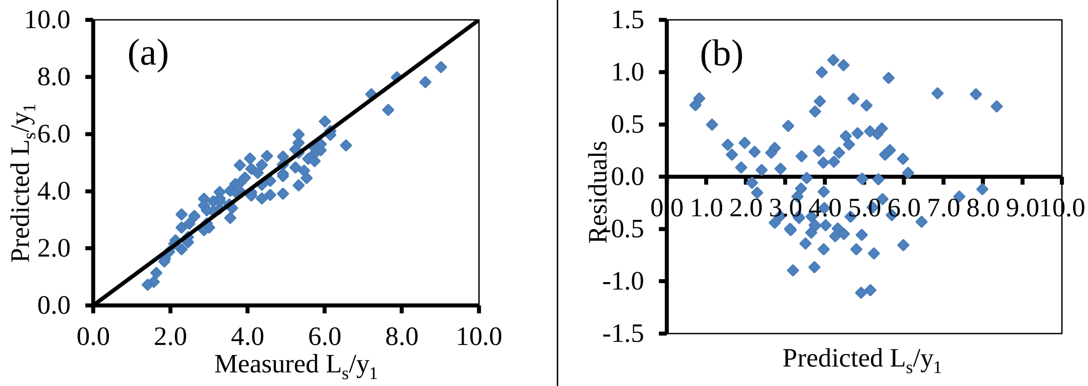

Figure 11 Comparison between the measured and predicted data for all experimental data of Ls/y1 for Eq. (4)

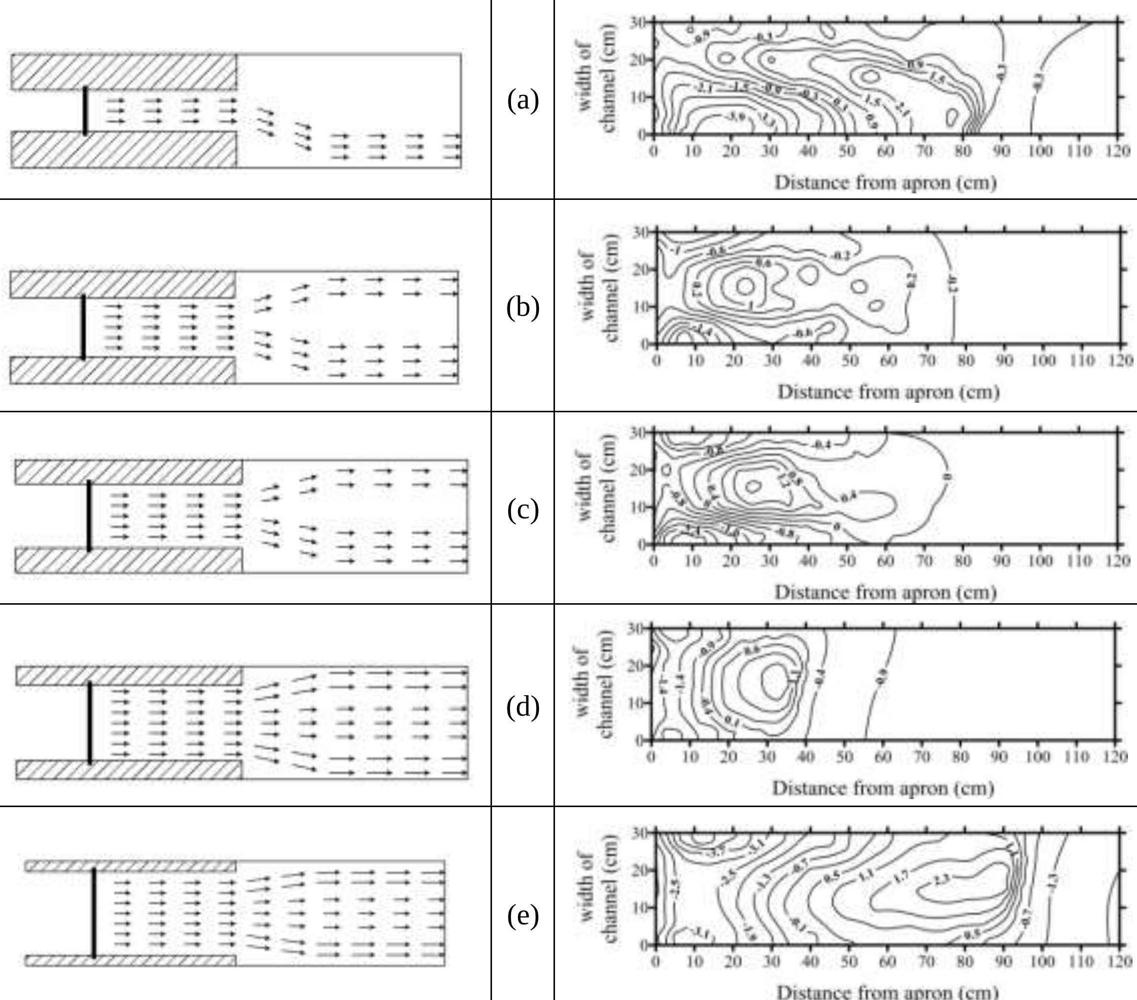

Figure 12 Flow and scour patterns for different expansion ratio e, at F_1 = 5.0 (a) e = 2.73 (b) e = 1.92 (c) e = 1.76 (d) e = 1.50 (e) e = 1.25

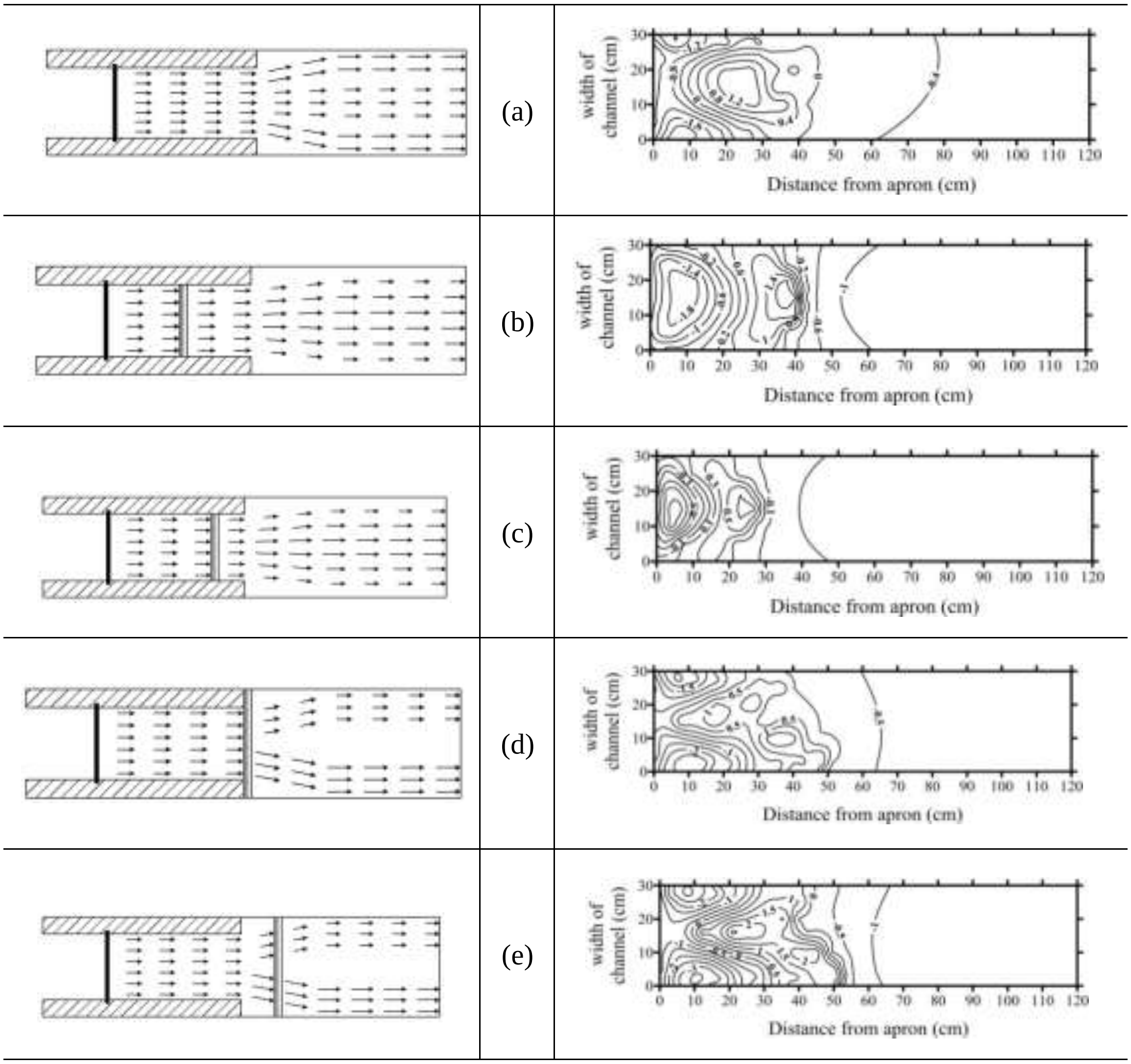

Figure 13 Flow and scour patterns for different Bucket sill position L_B/L_A, at F_1 = 5.9 (a) No sill case (b) L_x/L_B = 0.20 (c) L_x/L_B = 0.30 (d) Lx/L_B = 0.40 (e) L_x/L_B = 0.50