Gravimetric geoid for Egypt based on properly application of Helmet's method of condensations with window remove-restore technique

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Cover Page Footnote
The authors are grateful for the careful reviews of the anonymous reviewers. Indeed, it helped us to reconsider some essentials which lead to some additional test results and to a considerable (major) improvement of the paper. They also express their gratitude to the editor-in-chief.
Gravimetric Geoid for Egypt Based on Proper Application of Helmet's Method of Condensations with Window Remove-restore Technique

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Abstract

This study uses the window method for merging the gravity field wavelengths inside the remove-restore technique (RRT) to get the gravimetric geoid for Egypt using Helmert's models of condensation. In this case, the window approach (Abd-Elmotaal and Kühtreiber, 2003) was utilized to avert taking the topographic-condensation masses into account twice inside the DataFrame. A gravimetric geoid for Egypt has been calculated utilizing Helmert's first and second condensations approach and the Airy-Heiskanen model. Within the context of the geoid computation, a thorough comparison between the various Helmert approaches and Airy-Heiskanen has been made. The comparison uses the estimated geoid indicators before and after scaling to the GPS/leveling geoid as well as the residual gravity anomalies that remain after the removal step. The outcomes demonstrated that the window technique reduced gravity anomalies are the smooth, most objective, and have the narrowest range for all used gravity reduction techniques. In addition, the gravity anomalies that were reduced as a result of Helmert's first condensation method are nearly identical to those that were reduced using the Airy-Heiskanen technique. The results of the geoid computed through Helmert's first condensation and the Airy-Heiskanen technique are identical. Furthermore, although the indirect effect is minimal in the case of Helmert second condensation method, it is essential for determining the 1 cm geoid precision. Finally, geoid undulations computed from the Helmert second condensation method are better than those calculated from the other two methods.

Keywords: Gravimetric geoid for Egypt, Gravity anomaly data, Helmert's 1st and 2nd method of condensation, Window technique

1. Introduction

Using gravimetric data as the boundary value and solving a geodetic boundary value problem (GBVP), such as the Stokes or Molodenski problem, one can calculate the local geoid. The gravimetric technique for local or global gravimetric geoid determination refers to such a procedure. Both determining the global geoid and determining the localized geoids are impossible. The Stokes' integral solution can be improved by an alternative approach known as the modified Stokes' integral (Rstone et al., 1998). Gravimetric data, gravitational model, and Digital Height Model (DHM) are used as input observations (see, e.g. (Denker et al., 1986; Tziavos et al., 1992)). In this case, gravity anomalies and geoid undulations are divided into three parts known as short, medium, and long wavelength contributions. The long wavelength components are represented by a global geopotential model (GGM). This part can be calculated mathematically. The so-called residual gravity anomalies are obtained. The short and medium wavelength components are calculated using Stokes' integral with the residual gravity anomalies of the local area. By restoring the topographic masses and long wavelength components of geoid heights, final geoid heights can be attained. Here, it is worth pointing out that FFT or
FHT techniques can be used to solve the Stokes’ integral. The benefit of the remove-restore technique is that only the gravimetric observation data in a limited area is needed (Torge, 1991). So far, it has been extensively studied for precise determination of local geoid particularly in mountain regions (see, e.g. (Forsberg, 1991; Flury and Rummel, 2009; Sideris and Forsberg, 1990; Sideris and Li, 1992)).

The precision of such derived geoid heights depends on the precision of the three types of wavelength ingredients. Error sources and their effect on geoid determination using RRT are given in Li and Sideris (1994). Error in the long wavelength components is presented by the spherical harmonic coefficients. They cannot be removed using dense gravimetry data around the determination point. Error in medium and short wavelength components depends on the density and precision of the local gravity data, coverage, and spacing of the digital terrain model and improper modeling of the terrain. The relative accuracy of the three components has been given by Schwartz et al. (1987). It is clear that the most significant error results from the geopotential model. The remaining two errors are rather minor and can be reduced or avoidable by proper modeling of the topographic effect and the utilization of dense gravity anomalies and heights. It was concluded in the literature that the accuracy of a gravimetric geoid can be estimated by comparing geoid undulations with those obtained from GPS/leveling in an absolute or relative sense (e.g. (Denker and Wenzel, 1987; Mainville et al., 1992)).

It is widely acknowledged that the geoid is the equipotential surface of the gravity field at the mean sea level. It provides a consistent height system of topography on land and sea. As the majority of measurements used in geodesy are referred to the Earth’s gravity field, the determination of the geoid, the physical surface of the Earth, or the figure of the Earth becomes one of the major objectives of geodesy. Thus, it has been widely studied since the 1880s (Torge, 1991). The earliest definition of the geoid may date back to Gauss in 1828, Listing in 1873, and Helmert in 1880/1884 (Torge, 1991). From then, scientists have focused on their research interests on the determination of the geoid and its applications—geophysical interpretations (Bowin et al., 1986; Bowin, 1994; Chao, 1994; Fotiou et al., 1988; Hayling, 1994; Livieratos, 1994; Pick, 1994). Many methods have been developed for the determination of the local geoid. These methods can be categorized into three types: the geometric method, the gravimetric method, and a combination of various types of geometric and gravimetric data (the hybrid method). Astronomical, satellite altimetry, and GPS techniques are the direct methods. The geoid determination by solving the GBVP using gravity anomalies is known as the gravimetric method. The combined processing of gravimetric and geometric data can be performed by techniques, such as least-squares collocation (LSC), least-squares spectral combination (LSSC), and least-squares adjustment.

Since the geoid is the fundamental reference surface for the orthometric height of points, the accurate computation of the local geoid is of great significance (interest) for mapping and surveying. In recent decades, the geoid precision has been significantly improved ((Ayhan, 1993; Denker, 1991; Torge et al., 1989; Tscherning and Forsberg, 1986; Tziavos, 1987; Vaníček and Kleusberg, 1987); 1994 (Denker and Torge, 1993; Denker et al., 1994, 1995; Milbert, 1993; Sideris and She, 1995; Vaníček et al., 1995)); however, specific applications still require more precision to be introduced. Geometric leveling is commonly used to fix the heights of points on the surface of the Earth. However, many efforts have been carried out in recent decades to develop alternate techniques and technologies as it is highly demanding of labour and time. Currently, the relative positioning accuracy of the Global Positioning System (GPS) is a few millimeters plus 1–2 ppm. If the geoid has sufficient accuracy, GPS-derived ellipsoidal heights can be transferred to orthometric heights (GPS/leveling). The accurate local geoid determination will enable surveys to use GPS to its fullest capability, replacing geometric leveling. The probability of determining orthometric heights without leveling has been widely studied in recent decades. Examples may be found in Schwartz et al. (1987), Sideris (1993), and Engelis et al. (1985).

However, the determination of orthometric heights by GPS/leveling is not the only application of the geoid. The precise computation of local geoids makes it possible to study oceanography. The geoid surface can be used to determine sea surface topography and the moving features of sea currents (e.g. (Engelis et al., 1985; Nerem and Kobisly, 1994)). The sea surface topography (from the dynamic ocean surface to the geoid) is the signal that carries crucial information about the ocean circulation patterns.

The precise determination of local geoids is a challenge to geophysicists. Nowadays, geoid heights become routine data for geophysical interpretations. The geophysical applications of geoid cover (1) the upper crust density anomalies (Fotiou et al., 1988), (2) deep Earth mass anomaly structure (Bowin et al., 1986; Bowin, 1994; Hager, 1984), (3) strain and stress...
field (Livieratios, 1994; Dermanis et al., 1992; Livieratios, 1987), (4) tectonic forces (Pick, 1994), (5) oceanic lithosphere structure (Cazenave, 1994; Forsyth, 1985; Sandwell and Mackenize, 1989; Wunsch and Stammer, 1993), (6) rotation of the Earth (Chao, 1994), and (7) geophysical prospecting (Hayling, 1994). These methods are time-independent. In fact, the gravity field varies with time. Investigating the causes of the variations is of great interest to the geophysicists, seismologists, and oceanographers. Therefore, not only the geoid but also its variation needs to be precisely determined.

2. The data

2.1. Gravity data

The gravity anomalies data Fig. 1 illustrates irregular distribution with wide gaps, particularly on land. The coverage at the Red Sea is also good. The gravity data is extended over the region’s $19^\circ \leq \phi \leq 35^\circ N$ latitude and $22^\circ \leq \lambda \leq 40^\circ E$ longitude. There are 102419 stations, and they have gravity anomalies in the range of $[-210.6, -315.0]$ mgal. These points are irregularly distributed with many significant gaps. The marine data has been taken from the National Geophysical Data Center (NGDC) Marine Trackline Geophysics database. The land data has been provided by the National Gravity Standardization Base Net (NGSN77), Egyptian Survey Authority (ESA), and the General Petroleum Company (GPC).

2.2. GPS benchmarks

The present work uses the GPS data collection to validate the recommended methodologies, which consist of 30 GPS stations with known geoid undulations in Egypt. The overall total number of GPS points is too low compared with the land surface of Egypt, despite these stations being evenly dispersed across the whole nation (Fig. 2).

2.3. Digital height models

Determining potential and its first derivative of the topographic masses often call for a collection of fine and coarse digital terrain models. The effect of topographic/isostatic (T/I) or topographic/condensation (T/C) masses has been calculated using the TC program (Forsberg, 1984). The fine digital height model EGH13S03 3” × 3” and the coarse one EGH13S30 30” × 30” DHMs have been used (Abd-Elmotaal et al., 2013). They are covered the window $18.5^\circ \leq \phi \leq 35.5^\circ N$ and $21.5^\circ \leq \lambda \leq 40.5^\circ E$, Fig. 3.

3. Window-remove-restore technique (WRRT)

The attraction of T/I or T/C masses is often reduced from the free-air gravity anomalies and then restored to the resultant geoidal undulations using the RRT. In this situation, the reduced gravity anomalies are calculated by (Abd-Elmotaal and Kühtreiber, 2003) the equation

$$\Delta g_{\text{red}} = \Delta g_{F} - \Delta g_{GM} - \Delta g_{h}$$

(1)
where $DgF$ is the free-air gravity anomalies; $DgGM$ are the anomalies computed from the geopotential gravitational field; and $Dgh$ denotes the effect of T/I or T/C masses on gravity and will be studied in detail later. $DgGM$ can be determined from spherical harmonic coefficients (see (Heiskanen and Moritz, 1967)).

Then, as part of the conventional RRT, the final calculated geoid $N$ is provided by (Heiskanen and Moritz, 1967)

$$ N = N_{A} + N_{GM} + N_{h} $$

where $N_{GM}$ denotes the impact of the gravity anomalies after reduction; and $N_{h}$ denotes the indirect effect on geoid undulation (The removal or shifting of masses underlying the gravity reductions change the gravity potential and, hence, the geoid. This change of the geoid is an indirect effect of gravity reductions).

The geoid undulation determined from the geopotential gravitational model can be found in Heiskanen and Moritz (1967).

The gravimetric technique of geoid computation involves the evaluation of the Stokes’ integral which is given by (Abd-Elmotaal and Kühntreiber, 2003)

$$ N_{A} = \frac{R}{4\pi\gamma} \int_{\sigma} \Delta g_{red} S(\psi) \, d\sigma $$

where $N_{A}$ represents the geoidal undulation; $d\sigma$ refers to the integration’s surface element over the unit sphere; $R$ stands for the reference ellipsoid mean radius; $\gamma$ is the normal gravity of the reference ellipsoid; and $\Delta g_{red}$ refers to the reduced gravity anomaly on the gird. The geocentric angle $\psi$ is the angle between the radius vectors of the computation point $r_{p}(R, \varphi, \lambda)$ and the running point $r_{Q}(R, \varphi', \lambda')$ given by

$$ \cos \psi = \sin \varphi \sin \varphi' + \cos \varphi \cos \varphi' \cos (\lambda' - \lambda) $$

where $\varphi, \lambda$ represent the geodetic latitude and longitude of the computed station and $\varphi', \lambda'$ denote the geodetic latitude and longitude of the running element.

The Stokes’ function, $S(\psi)$, in Eq. (3), can be expressed as (Heiskanen and Moritz, 1967)

$$ S(\psi) = \frac{1}{\sin^{2}(\frac{\psi}{2})} - 6 \sin^{2}(\frac{\psi}{2}) + 1 - 5 \cos \psi - 3 \cos \psi \ln \left( \sin^{2}(\frac{\psi}{2}) + \sin^{2}(\frac{\psi}{2}) \right) $$

The conventional gravity reduction for the influence of the (T/I or T/C) masses is schematically depicted in Fig. 4. This process of eliminating the impact of the topographic and condensation masses’ impact has a theoretical issue. Since it is a part of the global reference field, some of the masses’ influence is double removed. This causes a portion of the T/I or T/C masses to be given more thought. This can be summarized in the following. For a point $P$ within the circle, the short-wavelength part based on the masses is calculated. The global masses, seen in Fig. 4 as a shaded rectangular area, are in charge of the long-wavelength ingredient of the Earth’s gravitational potential reference field. Removing their impact often entails removing their influence. The masses within the circle (which are double-hatched) are considered twice.

A potential solution to this problem is to modify the gravitational model for a fixed DataFrame to account for the influence of the masses. Schematically, Fig. 5 illustrates the benefit of the WRRT. Take an observation at point $P$. The short-wavelength component, dependent on the (T/I or T/C) masses, currently can be estimated using the masses of the entire data region (small rectangle). The T/I or T/C masses of the data window impact the reference...
field coefficients. Potential coefficients are subtracted to produce the modified reference field. Therefore, using this modified reference field to eliminate the long wavelength component, a portion of the T/I or T/C masses are not taken into account twice (no region with two hatches in Fig. 5).

Therefore, according to Abd-Elmotaal and Kühntreiber (2007), the removal step of the WRRT is as follows:

$$\Delta g_{\text{red}} = \Delta g_T - \Delta g_{\text{GMadapt}} - \Delta g_h$$

where $\Delta g_{\text{GMadapt}}$ is the adjusted reference field’s contribution. The WRRT restoration step is represented by the following syntax:

$$N = N_{\Delta g} + N_{\text{GMadapt}} + N_h$$

Here $N_{\text{GMadapt}}$ represents the contribution made by the modified reference field.

The gravimetric geoid determination after the window technique is summarized in Fig. 6.

4. Computation procedures

According to Heiskanen and Moritz (1967), the Newton integral for the potential of the terrain masses is written as follows:

$$V_p = G \iiint_{V_{\text{prism}}} \rho(Q) \frac{1}{r_{PQ}} dv$$

where $G$ is the Newton gravitational constant and $\rho$ is the topographic masses density or the difference in density between the terrain and water in ocean regions. The Digital Terrain Model (DTM) approximates the Earth’s surface in planar space. In this situation, the effects of T/I or T/C can be calculated in three dimensions (prism integration) or two dimensions (Gauss quadrature or the Gauss-Legendre Quadrature method). In the present research, Eq. (8) may be represented in the following way when DTM is thought of as prisms with constant density:

$$V_p = G \iint_{V_{\text{prism}}} \rho(Q) \frac{1}{r_{PQ}} dv$$

where $P$ denotes the computation point concerning a Cartesian coordinate system, and $Q$ denotes the source point for the same Cartesian coordinate system, with coordinates $x'$, $y'$, and $z'$ (see Fig. 7) and

$$r_{PQ} = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$$

is the Euclidean distance between Q and P with $\Delta x = x - x'$, $\Delta y = y - y'$ and $\Delta z = z - z'$.

Inserting Eq. (10) into Eq. (9) gives

$$V = G \iiint_{V_{\text{prism}}} \rho(Q) \frac{1}{\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}} d\Delta x' d\Delta y' d\Delta z'$$

The integral of Eq. (11) can be found in Ref (Nagy, 1966; Nagy et al., 2000):
\[ V = G \rho(Q) \left[ \| (\Delta x)(\Delta y) \ln((\Delta z) + r_{PQ}) + (\Delta x)(\Delta z) \ln((\Delta y) + r_{PQ}) + (\Delta y)(\Delta z) \ln((\Delta x) + r_{PQ}) \right] - \frac{(\Delta x)^2}{2} \arctan \frac{(\Delta y)(\Delta z)}{\Delta x r_{PQ}} - \frac{(\Delta y)^2}{2} \arctan \frac{(\Delta x)(\Delta z)}{\Delta y r_{PQ}} - \frac{(\Delta z)^2}{2} \arctan \frac{(\Delta x)(\Delta y)}{\Delta z r_{PQ}} \]

Also, the first derivative of Eq. (12) which denotes the attraction is given by (Nagy, 1966)

\[
\frac{\partial V}{\partial z} = G \rho(Q) \left[ \| (\Delta x) \ln((\Delta y) + r_{PQ}) + (\Delta y) \ln((\Delta x) + r_{PQ}) - \right.
\]

\[
- (\Delta z) \tan \left( \frac{(\Delta x)(\Delta y)}{\Delta z r_{PQ}} \right) \left| \begin{array}{c}
\gamma_1 \gamma_2
\end{array} \right|
\]

(13)

In our computations, the surface of the computation is assumed to be the plane for the prisms which lay near the computation point. The Earth’s curvature is considered for elements (prisms) that are far away from the point under consideration using the following equation:

\[ \text{Supel} = \frac{S^2}{2R} \] *(14)*

where \( S \) represents the distance between the point under consideration and the source point and \( R \) is the mean radius of Earth. The superelevation computed above gives the \( z \)-shift of the selected prism below the tangential plane. The value of superelevation is an approximate value, and it is valid for a few Kilometers only.

For the purpose of simplicity, let’s assume that the coordinate system’s origin is at \( P(x, y, z) \). If this is the case, coordinates defining the prism must be transformed using a simple 3D shift if the coordinate system’s orientation is unchanged.

According to Smith et al. (2001), Eq. (9) will be changed to the following equation to determine the condensation effect in the context of Helmert’s models of condensation:

\[ V = G \int_{P}^{k} \rho' \, d\sigma' \] *(15)*

where \( d\sigma = dx'dy' \) and \( k \) is the surface density which is the function of the topographic height (see Fig. 8). The surface density \( k \) can be determined from the constant density of the prism in linear approximation as

\[ k = \rho H \] *(16)*

This supports the theory of mass conservation based on a local mass balance inside any terrain column.

The impact of condensation masses on potential and the first derivative of the condensed masses may therefore be expressed as follows:

\[ V^c = G \rho(Q) \| (\Delta x) \ln((\Delta y) + r^*) + (\Delta y) \ln((\Delta x) + r^*) - \right. \]

\[ - z \arctan \left( \frac{(\Delta x)(\Delta y)}{(z - z^*) r_{PQ}} \right) \left| \begin{array}{c}
\gamma_1 \gamma_2
\end{array} \right| \]

(17)

\[ \frac{\partial V^c}{\partial z} = G \rho \Delta z \arctan \left( \frac{(\Delta x)(\Delta y)}{(z - z^*) r_{PQ}} \right) \left| \begin{array}{c}
\gamma_1 \gamma_2
\end{array} \right| \]

(18)

where

\[ r^* = \left( (x - x')^2 + (y - y')^2 + (z - z^*)^2 \right)^{1/2} \] *(19)*

and \( z_1 \) is the height of the computation point. In this case, \( \Delta z \) is equal to the orthometric height and \( z^* = H(x, y) \) in case of Helmert’s 1st approach of condensation and \( z = -D - H(x, y) \) in case of Helmert’s 2nd approach of condensation.

5. Topographic-condensation masses harmonic analysis

In the Helmert condensation reduction, the terrain masses are relocated along the local vertical and compressed on a parallel surface located 21 km (the original surface of Helmert’s 1st condensation
model) below the geoid. The condensation layer in the current application is 32 km (generalized model, Heck (2003)) below the geoid (Helmert’s 1st condensation method). The masses are immediately condensed onto the surface of the geoid in the 2nd Helmert condensation reduction instance. With the coefficients (derived from Heck, 2003), the condensed compensated topography resulting from these models can be represented once or more as a series of spherical harmonics as follows:

\[
\begin{align*}
\text{coef}^{n,l}_{nm} &= \frac{3}{(2n+1)\bar{\rho}} \left( 1 - \left( \frac{R_c}{R} \right)^n \right) h a^{1a}_{nm} \\
&\quad + \frac{n + 2}{2} \left( \frac{R_c}{R} \right)^n h a^{2a}_{nm} + \frac{1}{3} \left( \frac{n + 2}{2}\right) \frac{(n + 2)(n + 1)}{2} \\
&\quad - \left( \frac{R_c}{R} \right)^n h a^{3a}_{nm} + \frac{1}{n + 3} \sum_{k=4}^{n+3} \left( \frac{n + 3}{k} \right) \cdot h a^{k}_{nm} \\
\end{align*}
\]

where

\[
ha^{k}_{nm} = \frac{1}{4\pi} \int_{-\pi}^{\pi} \left( \rho(Q) \right) \left( \frac{H_i(Q)}{R} \right)^k \gamma_{nm} d\sigma
\]

\[
R_c \quad \text{represents the radius of the (approximate) condensation sphere; it can be identified as } R_c = R - 32 \text{ km in the case of Helmert’s 1st condensation method, while it holds } R_c = R \text{ in the case of Helmert’s 2nd method. } \rho_c \quad \text{represents the crust density, and } \bar{\rho} \quad \text{is the mean density of the Earth. If the topographic heights of the Earth’s surface are stated in terms of their corresponding rock heights, the aforementioned equations can be simplified (Rummel et al., 1988). Planar approximation allows the following expression to be used:}
\]

\[
H_{eq} = \begin{cases} 
\text{land : } H_{eq} = H(Q) \\
\text{ocean : } H_{eq} = \frac{\rho - \rho_w}{\rho} H(Q)'
\end{cases}
\]

6. Gravity reduction computations

Egypt’s residual gravity anomalies and geoid heights were conducted using the following parameter set:

\[
D_1 = 30 \text{ km} \\
\rho_{cr} = 2.67 \text{ g/cm}^3
\]

The geopotential model GO_CONS_GCF_2_TIM_R3 (Pail et al., 2011) has been used up to degree and order 250 to implement the conventional RRT. An adapted reference field has been generated by eliminating the harmonic coefficients of the T/I or

T/C masses of the data window determined by Eq. (20) from the GO_CONS_GCF_2_TIM_R3 coefficients. The WRRT has been applied using this adjusted reference field. The statistics for gravity reduction for conventional and WRRT for various gravity reduction strategies are shown in Table 1 after each reduction phase. It is worth noting that the reduced anomalies are approximately the same for all different gravity reduction techniques. The window strategy provides the best minimized gravity anomalies, as Table 1 demonstrates. The standard deviation has decreased by nearly 30%, and the range has shrunk by one-third. In addition, the reduced anomalies are centered and impartial. Because of this characteristic, window-technique reduced anomalies are especially well suited for all geodetic applications and perform effectively in them.

7. Geoid computations

In the current study, three different methods have been utilized to calculate the gravimetric geoid for Egypt considering WRRT. Those are:

(1) Window geoid for Airy-Heiskanen model
(2) Window geoid for Helmert’s 1st model of condensation.
(3) Window geoid for Helmert’s 2nd model of condensation.

The information of geoid undulation for the three approaches is displayed in Table 2.

Every calculated geoid is contrasted with a GPS/leveling geoid. First, the polynomial structure of the

<table>
<thead>
<tr>
<th>Reduced gravity</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Sdv</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta g_{Airy})</td>
<td>-210.6</td>
<td>315.0</td>
<td>-27.6</td>
<td>50.6</td>
</tr>
<tr>
<td>(\Delta g_{Heq} - \Delta g_{GM} - \Delta g_{Airy})</td>
<td>-96.7</td>
<td>138.0</td>
<td>14.7</td>
<td>27.4</td>
</tr>
<tr>
<td>(\Delta g_{Heq} - \Delta g_{GM} - \Delta g_{Heq_{win}})</td>
<td>-96.2</td>
<td>137.3</td>
<td>14.6</td>
<td>27.2</td>
</tr>
<tr>
<td>(\Delta g_{Heq} - \Delta g_{GM} - \Delta g_{Heq_{win2}})</td>
<td>-71.8</td>
<td>131.1</td>
<td>10.17</td>
<td>29.1</td>
</tr>
<tr>
<td>(\Delta g_{Airy_{win}})</td>
<td>-93.7</td>
<td>117.3</td>
<td>-4.8</td>
<td>21.3</td>
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<tr>
<td>(\Delta g_{Airy_{win}} - \Delta g_{GM_{adpt}} - \Delta g_{Airy_{win}})</td>
<td>-93.45</td>
<td>116.9</td>
<td>-4.75</td>
<td>21.2</td>
</tr>
<tr>
<td>(\Delta g_{Airy_{win}} - \Delta g_{GM_{adpt}} - \Delta g_{Heq_{win}})</td>
<td>-89.5</td>
<td>114.5</td>
<td>-5.22</td>
<td>22.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Geoid technique</th>
<th>Statistical parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airy-Heiskanen model</td>
<td>7.11</td>
</tr>
<tr>
<td>Helmert’s first method of condensation</td>
<td>7.32</td>
</tr>
<tr>
<td>Helmert’s second method of condensation</td>
<td>7.65</td>
</tr>
</tbody>
</table>
differences in absolute geoid between the GPS/leveling geoid and traditional RRT (Helmert’s first method for Example) are shown in Fig. 9. The absolute geoid discrepancies range between $-2.8 \, \text{m}$ and $9.2 \, \text{m}$ with an average of $1.8 \, \text{m}$ and standard deviation of $3.0 \, \text{m}$. The large-value residual has resulted actually from using the GPS data to determine the geoid. But these absolute values can be minimized when using a proper technique of best fitting for geoid.

Second, Fig. 10 indicates the absolute geoid discrepancies between the GPS/leveling geoids and the window approach utilizing the same method of gravity reduction (Helmert’s first method of condensation). The polynomial structure for the differences in Fig. 10 is better than it was in the case of the conventional RRT, Fig. 9. The standard deviation decreases to around $20 \, \text{cm}$, and the range of discrepancies to about $1.2 \, \text{m}$. One can see that the value ranges between $-3.4 \, \text{m}$ and $7.7 \, \text{m}$ with an average of $0.6 \, \text{m}$ and standard deviation of $2.8 \, \text{m}$.

Furthermore, Table 3 displays the information on the absolute geoid variances between the determined geoids in the present study and the GPS/leveling geoid. It can be observed that the window remove-restore technique gives approximately the same differences to the GPS/leveling geoid (in terms of either the mean difference or the range/standard deviation) for the three gravity reduction techniques used in this investigation.

### Table 3. Statistics of the residuals between the calculated geoids and the GPS/leveling geoid for all different gravity reduction techniques. Units are in (m).

<table>
<thead>
<tr>
<th>Geoid technique</th>
<th>Statistical parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
</tr>
<tr>
<td>Traditional remove-restore technique</td>
<td>$-2.8$</td>
</tr>
<tr>
<td>Window technique for Airy-Heiskanen model</td>
<td>$-3.4$</td>
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<tr>
<td>Window technique for Helmert’s 1st method of condensation</td>
<td>$-3.3$</td>
</tr>
<tr>
<td>Window Technique for Helmert’s 2nd method of condensation</td>
<td>$-3.2$</td>
</tr>
<tr>
<td>GO_CONS_GCF_2_TIM3</td>
<td>$-6.7$</td>
</tr>
</tbody>
</table>

8. Conclusions

The combination of the geoid wavelengths cannot be handled appropriately by the traditional Stokes’ method with an unmodified Stokes’ kernel in the remove-restore scheme. In this research, the WRRT is applied to three models of gravity reduction techniques. These are the Airy–Heiskanen model and the Helmert’s first and second scheme of condensations. The geopotential model GO_CONS_GCF_2_TIM3 has been used for dealing with the long wavelength contribution to gravity anomalies and geoid determination. This research comprises two main objectives; in the first objective, the necessary theoretical backgrounds have been studied. Then, gravity reduction techniques have been used for geoid determination for Egypt. For performing the computation, the TC program (Forsberg, 1984) has been used after modification to calculate the attraction of condensation masses for Helmert’s first and second method of condensation. The condensation surface is taken to be $32 \, \text{km}$ below the mean sea level in the case of Helmert’s first method of condensation. It has been presumed that the crustal density is constant and equal to $2.67 \, \text{g/cm}^3$.

The results of the first step in our computations...
revealed that the reduced anomalies using the window approach are smooth and oscillating around zero for the three methods of gravity reduction. These anomalies can be used with more efficiency to determine the geoid and can be used in geophysical interpolation. Also, the geoid determined from the three methods of gravity reduction is approximately the same (there are some minor differences). This means that the WRRT handles the gravity reduction for all gravity reduction techniques properly. Furthermore, although the indirect effect is very small for Helmert’s 2nd method of condensation, it is very important for determining the 1 cm geoid level. Finally, geoid undulations computed from Helmert’s 2nd condensation method are better than those computed from the other two methods. The results of this study confirm that, in principle, all gravity reductions are equivalent and should yield the same geoid if the indirect effect has been considered, and also the indirect effect should be as small as possible. In addition, the anomalies resulting from this research fulfill the requirements (the anomalies are small and smooth and oscillate around zero), which should be considered when deciding to apply a reduction method to compute the geoid. Also, the results from this investigation may also be used to simulate the Earth’s internal gravity using geophysical inversion producers to determine the terrain’s true density. Finally, by the indirect effect and by proper application of the gravity reduction technique, there are some minor differences between geoid undulations computed from different kinds of gravity reduction techniques. Such differences may come from the lack of gravity data in land areas.

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References


Author’s contribution

The corresponding author is responsible for investigating the field of the paper and writing the paper and helping the third author with the programming and computations.

The second author is responsible for providing the data, discussing the results, and revision.

The third author is responsible for programming and software computation and helping with the writing of the manuscript.

Conflicts of interest

All co-authors have seen and agree with the contents of the manuscript and there is no financial interest to report. We certify that the submission is original work and is not under review at any other publication.
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تحيين سطح الجيودي الجرايفومترعي لمصر باستعمال طرق Helmert و Airy / Heiskanen للتكيف الطبوغرافي اعتمادا على تقيية الحاف وإضافة ذو النافذة

تستخدم هذه الدراسة طريقة النافذة لدمج أطوال موجات مجال الجاذبية داخل تقيية الحاف وإضافة Helmert التي تم استخدامها لنتيجة النافذة تحدث بواسطة (عبد المتالك و كثيري، 2003). لتجنب أي تأثير على الكتافط الطبوغرافية في الاعترافات من خلال تحميل الجيودي الجرايفومترعي للمصر باستعمال نهج Helmert الأول والثاني ونموذج Airy / Heiskanen ونموذج Helmert، ظهرت نتائج طبقية ونماذج Helmert المختلفة و نماذج Airy / Heiskanen التي تم استخدامها لجيودي المغدورة قبل وبعد التحويل إلى جيودي GPS. النماذج الطبوغرافية بالنسبة إلى شروط الجيودي المغدورة التي تقبل بعد خطوة الإزالة. أظهرت النتائج أن تقييم النواحي التي تقبل النماذج في الجيودية في أكثر سلاسة وأقل التعرية وليست أضيق نطاق لمجتمع تقييمات تقليل الجيودية المختلفة. بالإضافة إلى ذلك، فإن نماذج Airy / Heiskanen تحقق نتائج تقليل الجيودية الملائمة مع ذلك النواحي التي تقبلها باستخدام نموذج Helmert، النماذج الطبوغرافية عبر تكنولوجيا Helmert الأولى ونماذج Airy / Heiskanen الأول ونماذج Helmert الثنائية. بالإضافة إلى ذلك، فإن التأثير غير المباشر صغير جدا في حالة طريقة النافذة Helmert الثنائية، إلا أنه مهم جدا للتحديد دقة الجيودي 1 مم.