Side Lobe Level Reduction and Array Thinning of Concentric Circular Antenna Arrays

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Abstract

This paper presents a new beamforming technique based on the hybrid combination of the convolution algorithm (CA) and the genetic algorithm (GA) for reducing side lobe level (SLL) and array thinning of concentric circular antenna arrays (CCAA), which is denoted as the C/GA technique. The CA determines the excitations of the elements, while the GA optimizes the radii of the circular arrays to adjust the half-power beamwidth (HPBW). For CCAA consisting of \( M \) uniform feeding circular arrays, we assume that there are \( K = 2M \) excitation coefficients that are distributed symmetrically around the array center and arranged in a \((K \times 1)\) vector. The excitation vector is convolved by itself to generate a \((2M^2)\) vector of synthesized excitations. To maintain the array size, only \( M\) odd, synthesized excitations or \( M\) even synthesized excitations are selected to feed the synthesized arrays. Both excitations yield a doubling-down decrease in the SLL but with wider HPBWs than the original pattern. Thereby, the GA optimizes the radii of the circular arrays to control the synthesized HPBWs. The synthesized array size is minimized by carrying out array thinning by turning off the outer circular arrays of the synthesized CCAA in conjunction with optimization of the radii of the ON arrays.

Keywords: Antenna array synthesis, Array thinning, Beamforming, Computer simulation technology (CST), Concentric circular antenna array (CCAA), Convolution algorithm (CA), Genetic algorithm (GA), Side lobe level (SLL)

1. Introduction

In recent years, a rise in the utilization of devices with wireless communication capabilities has been noticed due to technological innovations brought on by modern life, and this is concurrent with crucial criteria such as higher data rates and large bandwidth for 5G and future wireless applications in which a massive multi-input multi-output (MIMO) becomes a mainstream technology (Pant et al., 2022). As is well known, antenna arrays have a long history and are evolving in tandem with the advancement of information and electronic technology (Xiao et al., 2022), representing an essential part of recent wireless applications such as radar, mobile, military, satellite, and wireless communications (Bera et al., 2019). Therefore, it is essential to concentrate on the antenna array design and synthesis to improve its radiation pattern to comply with the basic specifications such as low side lobe level (SLL), narrow half power beamwidth (HPBW), and low dynamic range ratio (DRR). Also, it is required to improve the gain so we shall achieve better transmission characteristics (Taser et al., 2022). Low SLL is desired to make the interference and unwanted signals from other systems using the same frequency band that do not interfere with the communication systems (Mandal et al., 2010). On the other side, stronger directivity, which is crucial for long-distance transmission, requires a narrow HPBW. A lower SLL array does not provide a narrow HPBW, and vice versa, hence designing an antenna array with low SLL and narrow HPBW is
challenging (Collin, 1985; Ballanis, 2005). Therefore, it is impossible to simultaneously improve performance and uphold both antenna array criteria. The radiation pattern improvement can be achieved by controlling the parameters of the antenna array, such as its geometrical design, inter-element spacing, the antenna array elements’ excitation coefficients (amplitude and phase), and the radiation pattern of each element. Antenna arrays were given names like linear, elliptical, cylindrical, planer circular, and concentric circular depending on their geometrical design. A concentric circular antenna array (CCAA) is made up of many concentric circular rings, each of which has a certain number of antennas (Dessouky et al., 2006). In comparison to alternative antenna array types, the main lobe of circular antenna arrays can steer in all azimuth directions without affecting their bandwidth, making them the optimum configuration (Ioannides and Balanis, 2005). Where the mutual coupling effect caused by the element spacing is unwanted, this problem is solved by a novel design named the concentric circular antenna array with sufficient inter-element separation, which has been applied in beamforming for both narrowband and broadband applications, as well as direction-of-arrival estimates (Dessouky et al., 2006), whereas in the existence of both high- and low-noise levels, the performance of CCAAs is investigated using the direction of arrival (DOA) estimation technique (Das et al., 2021). With all-azimuth scanning, as CCAA is capable of, the beam pattern maintains its circular symmetry and is invariant for 360° of azimuthal coverage (MangoudMohab et al., 2014). Several progressive algorithms have been used in the design and synthesis of CCAA, as applied on a structure consisting of 3 rings of a nonuniform CCAA design having the set of (4, 6, 10) elements such as the ant lion optimizer (ALO) in Ref (Das Avishek Durbadal et al., 2019). This ALO algorithm has produced a reduction in SLL concerning uniformly excited and spaced 3-ring CCAA structures from −11.23 dB to −35.64 dB and narrowing the first null beamwidth (FNBW) to 79.92° taking into consideration the effect of the mutual coupling. Moth flame optimizer (MFO) in Ref (Das et al., 2018) reduced the SLL to −36.84 dB and the −3 dB beamwidth to 28.08° by performing optimization on both the current excitation for every antenna array element and the interelement spacing between the elements in each circle. In Ref (Dib, 2017), symbiotic organisms search (SOS) provided good results in SLL that reached −33.47 dB by only optimizing the current excitations of elements, and the distance between the adjacent elements is assumed to be constant being 0.55 λ, 0.606 λ, and 0.75 λ for the first, second, and third rings, respectively. The firefly algorithm (FA) in Ref (Sharaqa and Dib, 2014), which reduced the SLL to −33.20 dB is considered a good result as compared with the results obtained from using both the evolutionary programming (EP) algorithm in Ref (Mandal et al., 2010) and the biogeography-based optimization algorithm (BBO) in Ref (Dib and Sharaqa, 2014a).

For lowering the size of uniform planar antenna arrays (UPAA) and its SLL, newly suggested hybrid beamforming algorithms are described in Ref (Elkhawaga et al., 2022). They utilized a hybridization of GA with two-dimensional (2D) convolution. Controlling the elements’ excitations and separation results in the SLL decrease. A novel method for decreasing SLL and rejecting interference signals in concentric hexagonal antenna arrays (CHAA) was introduced in Ref (Akhittha and Ram, 2022) using the differential evolutionary (DE) algorithm. The CHAA was chosen for the suggested technique because, in comparison to circular and linear antenna arrays, it would provide a lower SLL, which is seen to be an advantage of the technique. To make use of the unique geometrical properties of fractals that enable the multiband and wideband operation and suppress the emergence of grating lobes, the Eisenstein fractal array was introduced in Ref (El-Khamy et al., 2022). The proposed antenna array was thinned with the GA optimization approach to lower the high SLL experienced at large-scale arrays. To achieve the least SLL without compromising the array radiation pattern’s directivity, the ideal set of ‘on’ and ‘off’ antenna elements was found. For SLL reduction and thinning of elliptical cylindrical antenna arrays (ECAA) in radar systems, new hybrid array beamforming approaches are presented in Ref (Dawood et al., 2021). These techniques are based on the integration of the virtual antenna array (VAA) idea, PSO methodology, and hyper-beamforming. The VAA breaks down the ECAA to produce a linear antenna array (LAA) and elliptical antenna array (EAA). The PSO is used to construct effective patterns with low SLL by optimizing the number of antenna elements, element spacing, and excitations of the produced LAA and EAA. Hyper-beamforming was used to reduce SLL even more.

However, using the fewest possible antenna array elements for antenna array synthesis to achieve the desired radiation pattern characteristics is known as antenna array thinning, which can be performed by turning off several active elements in the antenna array, and the main motivations behind antenna array thinning are a reduction in system payload, price, weight, and power consumption. For array
thinning, a variety of optimization techniques have been used, such as the binary salp swarm algorithm (BSSA) introduced in Ref (Mondal and Saxena, 2019). The BSSA is used to determine the group of antennas that should be activated to produce a beam with the lowest SLL, and the number of switched-off elements equals about 235 elements from the total number of 440 elements distributed in 10 concentric rings producing SLL equal to −28.42 dB. Improved Binary Invasive Weed Optimization Algorithm (IBIWO) used in Ref (Wu et al., 2015), with the same CCAA configuration used in the CSA algorithm but in addition to performing optimization on the inter-element spacing between elements in the ring, the number of switched-off elements is 231 and the SSL reached −31.7 dB with maintaining the HPBW to be the same as HPBW obtained from a fully populated configuration. It is noticed that the previous two algorithms have introduced good results whether in the number of antenna array elements that can be switched off or the value of SLL as compared with the rest of algorithms like the firefly algorithm (FA) used in Ref (Sharaqa and Dib, 2014), biogeography-based optimization (BBO) used in Ref (Singh and Kamal, 2012), and the teaching–learning-based optimization (TLBO) presented in Ref (Dib and Sharaqa, 2014b).

In this paper, a state-of-the-art hybrid beamforming technique is introduced in this research for synthesizing CCAAs with a focus on achieving optimal SLL reduction and array thinning. This novel method, denoted as the C/GA technique, effectively combines the linear convolution algorithm with the powerful GA optimization. The primary goal of SLL reduction is attained by carefully adjusting both the element excitations and interelement spacing within the array. The linear convolution algorithm is used to determine the element excitations, while the GA optimizes the interelement spacing along the perimeters of the circles, thus effectively controlling the changes in the half-power beamwidth (HPBW) of the array pattern. A remarkable twofold reduction in SLL is achieved by convolving the excitation vector with itself just once and subsequently feeding the synthesized array elements with either even or odd excitations. However, it is important to note that this improvement comes with a relatively higher dynamic range ratio (DRR). In addition to SLL reduction, the proposed technique facilitates efficient array thinning. Specifically, antenna elements in the outer circles of the CCAA, characterized by low excitation currents, are systematically deactivated. Consequently, the overall antenna array size is significantly minimized, leading to a substantial decrease in DRRs, all while ensuring that the SLL remains lower than that observed in UCAAs. This approach represents a significant advancement in the field of antenna array synthesis and optimization. The computer simulation technology (CST) microwave software package is used to test the practical validation of the proposed technique using half-wave-length dipole antenna elements. The CST software is considered as a realistic testing environment for the synthesized CCAAs while considering the mutual coupling between the array elements.

The novelty of the proposed C/GA technique can be summarized as follows:

1. However the CCAA geometry is a popular type of planar antenna array, its current excitations cannot be represented or arranged in a square or rectangular matrix form. Due to the lack of a definite matrix form, it is challenging to apply the convolution theory to the current excitations. Therefore, the challenge was in how to apply the convolution theory to the CCAA geometry with minimum signal processing burden and complexity.

2. Based on the reality that each circle in the CCAA is fed by a uniform current excitation, we found that it would be simpler to convert the CCAA into an LAA, whose current excitations can be expressed in a one-dimensional matrix or a vector as explained later. Thus, it becomes possible to apply the convolution theory to the excitations to synthesize a new set of excitations that achieve the desired SLL reduction.

2. Geometry and array factor of CCAA

In a concentric circular antenna array, all antenna elements are organized in several concentric rings on the X–Y plane. Fig. 1 depicts the general configuration of the CCAA with M concentric rings, where the \( m \)-th \((m = 1, 2, ..., M)\) the ring has a radius \( r_m \) and the corresponding number of elements is \( N_m \). Assuming that each element of the array represents an isotropic source, then the array factor that can be used to characterize the beam pattern is given as follows:

\[
AF(\theta, \phi) = \sum_{m=1}^{M} \sum_{n=1}^{N_m} \hat{z}_{mn} \exp[jk r_m \left( \cos \phi n U + \sin \phi n V \right)]
\]  

(1)
\[ \begin{align*}
U &= \sin \theta \sin \varphi - \sin \theta_0 \sin \varphi_0 \\
V &= \sin \theta \cos \varphi - \sin \theta_0 \cos \varphi_0
\end{align*} \tag{2} \]

The pair \((\varphi_0, \varphi)\) indicates the steering direction, and the pair \((\theta, \varphi)\) indicates the arrival direction, and \(\varphi \) and \(\theta\) are the azimuth and elevation angle, respectively. 

\(k = \frac{2\pi}{\lambda}\) is the wave number, 
\( r_m = N_m d_m/2\pi\) is the radius of the \(m\text{th}\) ring, 
\( d_m\) is the interelement spacing of the \(m\text{th}\) ring, and  
\(\varphi_{mn} = 2\pi(n - 1)/N_m\) is the angular position of the \(n\text{th}\) element of the \(m\text{th}\) ring. Finally, \(\bar{x}_{mn}\) is the excitation amplitude of the \(n\text{th}\) element on the \(m\text{th}\) ring. In case each ring has a uniform excitation, the excitation coefficients of \(M\) rings can be arranged in an \((M \times 1)\) vector named as \(\bar{x}_M\), which can be expressed as 
\(\bar{x}_M = [I_1 I_2 \ldots I_M]^T\), for \(m = 1, 2, 3, \ldots, M\).

3. Proposed C/GA beamforming technique

In this section, we present our proposed C/GA beamforming technique, designed to effectively reduce the SLL of CCAAs. This approach represents a hybrid methodology that combines the linear convolution method with the optimization power of GA. To begin, we consider a cross-sectional view along the center of the CCAA construction, consisting of \(M\) uniform feeding rings, as depicted in Fig. 2. For the case of uniform feeding with excitation coefficients \(\bar{x}_M = [I_1 I_2 \ldots I_M]\), we obtain a \((2M \times 1)\) vector representation of the excitation coefficients, as illustrated in Fig. 2. The \(K = 2M\) excitation coefficients are symmetrically positioned around the array center and are arranged in a \((K \times 1)\) vector denoted as \(A_K\). This vector can be referred to as 
\(A_K = [I_{M-1} I_2 I_1 I_2 I_3 \ldots I_M]^T\) \tag{3}.

In general, the linear convolution of two vectors \(X_{M \times 1}\) and \(Y_{N \times 1}\) can be expressed as follows:
\(C_{I \times 1} = X_{M \times 1}^* Y_{N \times 1}\) \tag{4}.

---

**Fig. 1.** The concentric circular antenna array geometrical configuration.

**Fig. 2.** The cross-section of the geometry of the concentric circular antenna array and the resultant excitation vector.
where $C_{i+1}$ is the resultant convolution such that $(I = M + N - 1)$ whose elements $C(i, 1)$ are given by

$$C(i, 1) = \sum_{w=w_{\text{min}}}^{w_{\text{max}}} X(w, 1) Y(i - w + 1, 1)$$  \hspace{1cm} (5)

where $1 \leq i \leq M + N - 1$ and $w$ ranges over all values of $X(w, 1)$ and $Y(i - w + 1, 1)$, and the upper and lower limits of $w$ are given by

$$w = \begin{cases} \max(1, i - K+1) \\ \min(i, M) \end{cases}$$  \hspace{1cm} (6)

Accordingly, $w_{\text{min}} = \min(w)$ and $w_{\text{max}} = \max(w)$.

In the proposed C/GA technique, we have to convolve the $(K \times 1)$ excitation vector $A_{K \times 1}$ by itself such that the resultant vector $C_{(2K-1) \times 1}$ can be expressed as

$$C_{(2K-1) \times 1} = A_{K \times 1} \ast A_{K \times 1}$$  \hspace{1cm} (7)

Consequently, the proposed C/GA technique results in a significant twofold reduction in the SLL. To further elucidate this principle, let us consider the concept of the Fourier transform, which posits that the current distribution’s Fourier transform corresponds to the antenna array’s array factor. Mathematically, this relationship is expressed as follows:

$$AF(\theta, \emptyset) = F[A_{K \times 1}]$$  \hspace{1cm} (8)

where $F[.]$ denotes the Fourier transform.

By performing the Fourier transform on Equation (7), we obtain the following expression:

$$F[C_{(2K-1) \times 1}] = F[A_{K \times 1} \ast A_{K \times 1}]$$  \hspace{1cm} (9)

$$F[C_{(2K-1) \times 1}] = F[A_{K \times 1}] \ast F[A_{K \times 1}]$$  \hspace{1cm} (10)

Consequently, the synthesized array factor $AF_S(\theta, \emptyset) = F[C_{(2K-1) \times 1}]$ having the current excitations $C_{(2K-1) \times 1}$ can be considered as

$$AF_S(\theta, \emptyset) = [AF(\theta, \emptyset)]^2$$  \hspace{1cm} (11)

As evident from Equation (11), it is apparent that the synthesized array factor $AF_S(\theta, \emptyset)$ achieved using the excitation vector $C_{(2K-1) \times 1}$ is equivalent to the original array factor’s square $AF(\theta, \emptyset)$. This results in a twofold reduction in SLL when the normalized array factor’s side-lobe amplitudes’ fractional values are squared. However, it should be noted that the length of the resultant excitation vector $C_{(2K-1) \times 1}$ from the convolution process is considerably larger than the length of the original excitation vector $A_{K \times 1}$. To preserve the original array size while obtaining the synthesized array factor $AF_S(\theta, \emptyset)$, it becomes necessary to divide the excitation vector $C_{(2K-1) \times 1}$ into two distinct vectors, namely $c_{(K-1) \times 1}$O, which includes the odd excitations required to implement $AF_{SO}(\theta, \emptyset)$, and $c_{(K-1) \times 1}$E, which includes the even excitations required to implement $AF_{SE}(\theta, \emptyset)$. This partitioning allows for maintaining the original array size while effectively synthesizing the desired array factor and achieving the twofold SLL reduction.

The determination of the odd excitation vector can be accomplished using the following procedure:

$$c_{(K-1) \times 1}O = c_{0}(m, 1)$$

$$c_{0}(m, 1) = C(2i-1, 1)$$  \hspace{1cm} (12)

where $1 \leq (i = m) \leq \left(\frac{k}{2}\right)-1$

This excitation vector $c_{(K-1) \times 1}O$ will have $K = 2M$ excitation current amplitudes that are placed symmetrically around the array center. Then the $M$ excitations of $M$ rings can be written as follows:

$$X_M(m, 1) = I_O(m, 1) = c_{0}(m, 1)$$  \hspace{1cm} (13)

The even excitation vector can be determined as follows:

$$c_{(K-1) \times 1}E = c_{E}(m, 1)$$

$$c_{E}(m, 1) = C(2i, 1)$$  \hspace{1cm} (14)

where $1 \leq (i = m) \leq \left(\frac{k}{2}\right)-1$

This excitation vector $c_{(K-1) \times 1}E$ will have $K = 2M - 1$ excitation current amplitudes that are placed symmetrically around the array center. Then the $(M - 1)$ excitation coefficients of the $(M - 1)$ rings can be expressed as follows:

$$X_{M-1}(m, 1) = I_E(m, 1) = c_{E}(m, 1)$$  \hspace{1cm} (15)

Consequently, the synthesized array factor achieved using the odd excitation vector can be expressed as follows:

$$AF_{SO}(\theta, \emptyset) = \sum_{m=1}^{M} \sum_{n=1}^{N} I_O(m, 1) \exp[jk r_m (\cos \varphi_{mn} U + \sin \varphi_{mn} V)]$$  \hspace{1cm} (16)

In contrast, the synthesized array factor uses an even excitation vector, which can be written as
\[ AF_{SE}(\theta, \varnothing) = \sum_{m=1}^{M-1} \sum_{n=1}^{N} I_f(m, 1) \exp[jk(\cos \varphi_{m1} U + \sin \varphi_{m1} V)] \]

Thereby, \( AF_{SO}(\theta, \varnothing) \) is the array factor of the synthesized CCAAs\( SO \) using odd excitations that are distributed over \( M \) concentric rings, while \( AF_{SE}(\theta, \varnothing) \) is the array factor of the synthesized CCAAs\( SE \) using even excitations that are distributed over \((M-1)\) concentric rings.

The synthesized arrays, CCAAs\( SO \) and CCAAs\( SE \) achieve a twofold decrease in the SLL. However, it is important to note that the HPBW of these synthesized arrays is slightly wider than the HPBW of the original CCAA. To mitigate the changes in the synthesized HPBW, the GA is used to optimize the radii of the circular rings, denoted as \( r_{ms} \), which in turn controls the interelement spacing, represented as \( d_{m} \), within the range of \( 0.5\lambda \leq d_{m} \leq 0.9\lambda \). This range is chosen to avoid the occurrence of grating lobes. The optimization process aims to strike a balance between SLL reduction and maintaining the HPBW within acceptable limits, ensuring the overall effectiveness of the synthesized CCAA arrays. To achieve the optimization objective effectively, the GA minimizes the designed cost function \( CF \), which is defined as follows:

\[ CF = \min \left( \frac{|HPBW_{s} - HPBW|}{|SLL_{s} - SLL|} \right) \]

where \( d_{m} \) is optimized interelement spacing between elements on the perimeter of the \( m^{th} \) circle or ring and \( r_{ms} \) is the optimized radius of the \( m^{th} \) circle. HPBWs and SLLs are the synthesized half-power beamwidth and side lobe level, respectively. For the designed CF to be minimized, the absolute difference between the synthesized and original side lobe level \(|SLL_{s} - SLL|\) should be as great as possible, while the absolute difference between the synthesized and original half-power beamwidth \(|HPBW_{s} - HPBW|\) should be as small as possible.

4. **Discussion of simulation results**

In this section, we present several MATLAB simulation results and subsequent discussions about the performance of the proposed C/GA beamforming technique for SLL reduction in CCAAs. The effectiveness of the procedure is assessed through various performance metrics and comparisons with existing approaches. Simulations are conducted under diverse scenarios to gauge the robustness and adaptability of the proposed technique. Furthermore, we analyze the effects of several factors, including the number of elements, interelement spacing, and radii of circular rings, on the performance of the synthesized CCAA arrays. Moreover, we investigate the tradeoffs between SLL reduction and the widening of HPBWs to provide insights into the practical applications of the proposed method in real-world scenarios. The obtained simulation results are thoroughly discussed, highlighting the advantages and limitations of the C/GA technique. Moreover, comparisons are made with conventional beamforming methods, and the reasons for the observed improvements are expounded. The discussions shed light on the strengths of the proposed approach in achieving a twofold reduction in SLL while minimizing the changes in HPBWs through effective GA optimization. We also address potential challenges and areas for future research to further enhance the performance and applicability of the C/GA beamforming technique in diverse antenna array configurations.

4.1. **SLL reduction**

Consider a CCAA comprising \( 10 \) uniform rings without a central antenna element. The innermost ring consists of \( N = 3 \), and each subsequent ring has six more elements on the outside. The interelement spacing \( (d_{m}) \) within each ring is set at \( 0.5\lambda \), and the initial SLL is measured at \(-17.54\) dB. Upon applying the C/GA technique, the resultant excitation vector \( A_{K \times 1} = A_{19 \times 1} \) is divided into two sets of excitations \( I_{(M+1) \times 1} - o = I_{(10 \times 1) - o} \) containing the odd excitations and \( I_{(M+1) \times 1} - e = I_{(9 \times 1) - e} \) containing the even excitations. Use these sets to synthesize the array factor \( AF_{SO}(\theta, \varnothing) \) and \( AF_{SE}(\theta, \varnothing) \) respectively. The specific values of \( I_{(10 \times 1) - o} \) and \( I_{(9 \times 1) - e} \) are outlined in Tables 1 and 2, respectively. In Fig. 3, we illustrate the synthesized array factor \( AF_{SO}(\theta, \varnothing) \) alongside the original array factor \( AF(\theta, \varnothing) \), while Fig. 4 demonstrates the synthesized array factor \( AF_{SE}(\theta, \varnothing) \) compared with the original array factor \( AF(\theta, \varnothing) \). Besides, Tables 1 and 2 provide a comparative analysis of the synthesized array patterns concerning the uniform CCAA pattern, focusing on array size, interelement spacing, HPBW, SLL, and DRR. This analysis allows us to assess the performance improvements achieved through the C/GA technique in terms of array characteristics and radiation pattern qualities.

After analyzing the results outlined in Tables 1 and 2, it is obvious that both the odd and even excitations provide SLLs equal to \(-32.57\) dB and \(-31.72\) dB, respectively, which represent approximately a twofold decrease in the SLL compared with the SLL
Table 1. Comparison between the synthesized array factor $AF_{SO}(\theta, \phi)$ and the original array factor $AF(\theta, \phi)$ for concentric circular antenna arrays consisting of $M = 10$ uniform circles.

<table>
<thead>
<tr>
<th>Array factor</th>
<th>SLL</th>
<th>HPBW</th>
<th>Excitation currents</th>
<th>Inter element spacing</th>
<th>DRR</th>
<th>Array size</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AF(\theta, \phi)$</td>
<td>$-17.54 \text{ dB}$</td>
<td>$6.3^\circ$</td>
<td>[1,1,1,1,1,1,1,1,1,1]</td>
<td>$0.5 \lambda$</td>
<td>1</td>
<td>$M = 10$</td>
</tr>
<tr>
<td>$AF_{SO}(\theta, \phi)$</td>
<td>$-32.57 \text{ dB}$</td>
<td>$6.3^\circ$</td>
<td>[19,17,15,13,11,9,7,5,3,1]</td>
<td>$0.65 \lambda$</td>
<td>19</td>
<td>$M = 10$</td>
</tr>
</tbody>
</table>

Table 2. Comparison between the synthesized array factor $AF_{SE}(\theta, \phi)$ and the original array factor $AF(\theta, \phi)$ for concentric circular antenna arrays consisting of $M = 10$ uniform circles.

<table>
<thead>
<tr>
<th>Array factor</th>
<th>SLL</th>
<th>HPBW</th>
<th>Excitation currents</th>
<th>Inter element spacing</th>
<th>DRR</th>
<th>Array size</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AF(\theta, \phi)$</td>
<td>$-17.54 \text{ dB}$</td>
<td>$6.3^\circ$</td>
<td>[1,1,1,1,1,1,1,1,1,1]</td>
<td>$0.5 \lambda$</td>
<td>1</td>
<td>$M = 10$</td>
</tr>
<tr>
<td>$AF_{SE}(\theta, \phi)$</td>
<td>$-31.72 \text{ dB}$</td>
<td>$6.3^\circ$</td>
<td>[18,16,14,12,10,8,6,4,2]</td>
<td>$0.68 \lambda$</td>
<td>9</td>
<td>$M = 9$</td>
</tr>
</tbody>
</table>

4.2. Array thinning

Based on the previous results, array thinning, which involves reducing the number of elements in the synthesized arrays, has proven to be highly advantageous. It brings about several benefits, such as reducing the overall size of the array, simplifying the feeding network's layout, and ultimately lowering the cost of the entire array system. To perform array thinning, we focus on the synthesized array factor $AF_{SO}(\theta, \phi)$ and gradually deactivate entire antenna elements from the outside circles toward the array center. By selectively switching off the outer five circles, we achieve percentage reductions in the number of array elements, specifically 19%, 36%, 51%, 64%, and 75%. The corresponding SLL resulting from each level of thinning is documented in Table 3. Notably, even at 75%

Table 3. Percentages of array thinning of the synthesized array factor $AF_{SO}(\theta, \phi)$ indicating the resultant side lobe level and half power beamwidth.

<table>
<thead>
<tr>
<th>SLL</th>
<th>HPBW</th>
<th>Number of Switched off elements</th>
<th>Number of Switched off circles</th>
<th>Percentage of array thinning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-32.57 \text{ dB}$</td>
<td>$6.3^\circ$</td>
<td>0</td>
<td>0</td>
<td>0 %</td>
</tr>
<tr>
<td>$-30.66 \text{ dB}$</td>
<td>$6.3^\circ$</td>
<td>57</td>
<td>1</td>
<td>19 %</td>
</tr>
<tr>
<td>$-27.71 \text{ dB}$</td>
<td>$6.3^\circ$</td>
<td>108</td>
<td>2</td>
<td>36 %</td>
</tr>
<tr>
<td>$-24.78 \text{ dB}$</td>
<td>$6.3^\circ$</td>
<td>153</td>
<td>3</td>
<td>51 %</td>
</tr>
<tr>
<td>$-22.85 \text{ dB}$</td>
<td>$6.3^\circ$</td>
<td>192</td>
<td>4</td>
<td>64 %</td>
</tr>
<tr>
<td>$-18.1 \text{ dB}$</td>
<td>$6.3^\circ$</td>
<td>225</td>
<td>5</td>
<td>75 %</td>
</tr>
</tbody>
</table>
thinning, the SLL of the array remains lower than that of the original array, registering at $-18.1$ dB. In Fig. 5, we visualize the array thinning process applied to the synthesized $AF_{\text{SO}}(\theta, \varphi)$ pattern. Furthermore, Fig. 6 shows the tradeoff relationship between the percentage of array thinning and the associated SLLs. As expected, an increase in the thinning percentage leads to a corresponding increase in the SLL. This tradeoff is crucial to consider while optimizing array performance to strike an appropriate balance between array size reduction and maintaining the desired SLL.

Fig. 5. The array thinning of the synthesized $AF_{\text{SO}}(\theta, \varphi)$ pattern for thinning percentages: (a) 19 %, (b) 36 %, (c) 51 %, (d) 64 %, and (e) 75 %.
On the contrary, if we apply array thinning to the synthesized array factor $AF_{SE}(\theta, \varnothing)$, we gradually deactivate the outer four circles. As a result, the number of array elements is reduced by 36%, 51%, 64%, and 75%, respectively, as shown in Table 4. Remarkably, even at 75% thinning, the SLL of the array remains lower than that of the original array, measuring at $-18.1\, \text{dB}$. Fig. 7 illustrates the tradeoff between the percentage of array thinning and the corresponding SLL. As anticipated, an increase in the thinning percentage results in an elevation of the SLL. It is crucial to strike an optimal balance between array size reduction and maintaining the desired SLL levels when considering array thinning to optimize the overall array performance.

Table 4. Summary of the percentages of array thinning for the synthesized array factor $AF_{SE}(\theta, \varnothing)$, indicating the resulting side lobe level and half power beamwidth.

<table>
<thead>
<tr>
<th>SLL</th>
<th>HPBW</th>
<th>Number of switched off elements</th>
<th>Number of switched off circles</th>
<th>Percentage of array thinning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-29.73, \text{dB}$</td>
<td>$6.3^\circ$</td>
<td>108</td>
<td>2</td>
<td>36%</td>
</tr>
<tr>
<td>$-25.78, \text{dB}$</td>
<td>$6.3^\circ$</td>
<td>153</td>
<td>3</td>
<td>51%</td>
</tr>
<tr>
<td>$-23.34, \text{dB}$</td>
<td>$6.3^\circ$</td>
<td>192</td>
<td>4</td>
<td>64%</td>
</tr>
<tr>
<td>$-18.1, \text{dB}$</td>
<td>$6.3^\circ$</td>
<td>225</td>
<td>5</td>
<td>75%</td>
</tr>
</tbody>
</table>
From our point of view, the excitation current distribution of the antenna elements of the antenna array has the largest value in the middle of the array and decreases gradually as we move away from the center of the circles as shown in Fig. 8. Thus, the effect of turning off ‘thinning’ the entire elements of the outer circles is the best choice to reduce the DRR of the excitation currents. In addition to that the circles that are far from the center contain a large set of elements, so the

![Diagram](image)

**Fig. 9.** Dipole antenna: (a) dimensions, (b) H-plane pattern, and (c) E-plane pattern.
antenna array thinning by this technique makes it more executable.

5. Practical validation testing using CST simulations

In this section, the proposed technique is practically proved using the CST microwave studio software package considering the mutual coupling effect between the antenna elements, as it significantly affects the antenna radiation pattern in terms of the SLL and main beam characteristics. The synthesized CCAs are implemented using a \( \lambda/2 \) dipole element with a radius of 4.5 mm. Fig. 9 depicts the dipole element dimensions, H-plane pattern, and E-plane pattern. The dipole has a resonance frequency \( f_0 = 1 \) GHz, and this is indicated by the scattering parameter \( |S_{11}| \) versus the frequency as shown in Fig. 10.

5.1. CST implementation of \( M = 10 \) CCAA using the C/GA technique for SLL reduction

In this section, the practical performance of the proposed C/GA technique is validated by utilizing the CST microwave studio using a dipole antenna element instead of an isotropic antenna to verify the effect of mutual coupling. Consequently, the synthesized CCAs introduced in Section (4.1) are implemented utilizing the CST microwave studio. By applying the C/GA on the uniform \( M = 10 \) elements CCAA, the resultant \( (19/2) \) excitation vector \( A_{(19/2)} \) is divided into the \( I_{(M=10)-\alpha} = I_{(10/1)-\alpha} \) and \( I_{(M-1)-E} = I_{(9/1)-E} \) that contain odd and even excitations to implement the synthesized array factors \( AF_{SO}(\theta, \phi) \) and \( AF_{SE}(\theta, \phi) \), respectively. The geometrical structure of the original uniform \( M = 10 \) CCAA using the CST microwave studio is shown in Fig. 11. The implemented arrays using odd and even

![Fig. 10. The scattering parameter \( |S_{11}| \) versus frequency of the dipole antenna.](image)

![Fig. 11. The geometrical structure of the original \( M = 10 \) uniform concentric circular antenna arrays.](image)
Excitations are fed with the excitation coefficients and with uniform interelement spacing listed in Tables 1 and 2, respectively. The simulated 3-D radiation patterns of the implemented arrays are shown in Fig. 12, while the polar plots of the synthesized CCAAs are shown in Fig. 13. Table 5 shows a comparison between the original \( AF(\theta, \phi) \) and the synthesized CCAA patterns \( AF_{SO}(\theta, \phi) \) and \( AF_{SE}(\theta, \phi) \) using the CST microwave studio. By analyzing the results it is clear that the implemented patterns \( AF_{SO}(\theta, \phi) \) and \( AF_{SE}(\theta, \phi) \) provide the same SLL = –32 dB, which are approximately equal to the twofold decrease in the SLL of the uniform CCAA pattern \( AF(\theta, \phi) \), which equals –17.54 dB. Also, the synthesized patterns \( AF_{SO}(\theta, \phi) \) and \( AF_{SE}(\theta, \phi) \) provide the same HPBW = 6.2°, which are less than the original CCAA pattern that has a HPBW = 6.3°. When the simulation results from CST listed in Table 5 are compared with the simulation results from MATLAB as listed in Tables 1 and 2, it is clear that there is excellent compatibility between them, demonstrating the viability of the proposed SC/GA technique.

Fig. 12. The three-dimensional radiation patterns of (a) synthesized \((M = 10)\) concentric circular antenna arrays using C/GA for odd excitations and (b) synthesized \((M = 9)\) concentric circular antenna arrays using C/GA for even excitations.
6. Comparison with related work

In this section, the proposed SLL reduction technique is compared with some related state-of-art techniques such as a combination of ant lion optimizer (ALO) and sequential quadratic programming (SQP) (ALO-SQP) (Taser et al., 2022), firefly algorithm (FA) (Sharaqa and Dib, 2014), evolutionary programming (EP) (Mandal et al., 2010), symbiotic organisms search (SOS) (Dib, 2017),

Table 5. Comparison between the synthesized array factors $AF_{SO}(\theta, \varphi), AF_{SE}(\theta, \varphi)$ and the original array factor $AF(\theta, \varphi)$ for concentric circular antenna arrays consisting of $(M = 10)$ uniform circles using the CST microwave studio.

<table>
<thead>
<tr>
<th>Array factor</th>
<th>SLL</th>
<th>HPBW</th>
<th>Excitation currents</th>
<th>Inter-element spacing</th>
<th>DRR</th>
<th>Array size</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AF(\theta, \varphi)$</td>
<td>17.54 dB</td>
<td>6.3°</td>
<td>[1,1,1,1,1,1,1,1,1,1]</td>
<td>0.5 $\lambda$</td>
<td>1</td>
<td>$M = 10$</td>
</tr>
<tr>
<td>$AF_{SO}(\theta, \varphi)$</td>
<td>32 dB</td>
<td>6.2°</td>
<td>[1,3,5,7,9,11,13,15,17,19]</td>
<td>0.65 $\lambda$</td>
<td>19</td>
<td>$M = 10$</td>
</tr>
<tr>
<td>$AF_{SE}(\theta, \varphi)$</td>
<td>32 dB</td>
<td>6.2°</td>
<td>[2,4,6,8,10,12,14,16,18]</td>
<td>0.68 $\lambda$</td>
<td>9</td>
<td>$M = 9$</td>
</tr>
</tbody>
</table>

Fig. 13. The polar plot of the radiation patterns: (a) synthesized $(M = 10)$ concentric circular antenna arrays using C/GA for odd excitations and (b) synthesized $(M = 9)$ concentric circular antenna arrays using C/GA for even excitations.
Among the related techniques, it is clear that the ALO-SQP technique provides the lowest SLL and HBPW that equal $-28.26\, dB$ and $25.04^\circ$, respectively. However, when it is compared with the proposed technique, it is found that they provide approximately the same SLL, but the proposed technique provides a much narrower HBPW that equals $21.42^\circ$. And in terms of DRR, it is evident that the proposed technique provides the lowest DRR.

7. Conclusion

In this paper, a new beamforming method known as the ‘C/GA’ technique for SLL reduction and array thinning of circular concentric antenna arrays is presented. It is based on a hybrid combination of the convolution algorithm (CA) and the genetic algorithm (GA). While the GA optimizes the radii of the circular arrays to control the half-power beamwidth (HPBW), the CA calculates the excitations of the antenna elements. The C/GA provides a twofold decrease in the SLL of the array with minimum changes in the HPBW that can be readjusted using the GA. For a CCAA without a central antenna element that consists of $M = 10$ uniform rings, the innermost ring has three elements, while each succeeding ring has six more elements on the outside. The interelement spacing $d_m$ in any individual ring is $0.5\lambda$ and the SLL is $-17.54\, dB$. After applying the C/GA technique, it is clear that the odd and even excitations both yield SLLs equal to $-32.57\, dB$ and $-31.72\, dB$, respectively, which represent approximately a twofold decrease in the SLL compared with the SLL of the original CCAA that equals $-17.54\, dB$. Furthermore, by switching off the outer five circles gradually, the number of array elements is reduced by 19%, 36%, 51%, 64%, and 75%, respectively. It is worth noting that at 75% thinning, the SLL of the array equals $-18.1\, dB$ and is still lower than that of the original array.

The biogeography-based optimization (BBO) (Dib and Sharaqa, 2014a), and the chicken swarm optimization (CSO) (Ram et al., 2015) considering a uniform ($M = 3$) CCAA with uniform interelement spacing $d_m = 0.5\lambda$. These techniques are summarized in Ref (Taser et al., 2022) to synthesize an ($M = 3$) CCAA with sets of elements (8, 10, 12) in three circles from the innermost ring to the outer ring, respectively.

The proposed C/GA technique is applied to the ($M = 3$) CCAA with sets of elements (3, 9, 15) in three circles from the innermost ring to the outer ring, respectively, and with interelement spacing $d_m = 0.65\lambda$. The synthesized pattern using odd excitations $AF_{SO}(\theta, \phi)$ is compared with the uniform ($M = 3$) CCAA as shown in Fig. 14. The SLL of the synthesized pattern reached $-28.59\, dB$ while the SLL of the original array equals $-16.49\, dB$.

Table 6 shows a comparison between the proposed C/GA technique and the related techniques.
Declarations

Ethical Approval All authors agree to publish the research in this journal.

Declaration of competing interest

The authors declare no competing interests.

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